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Predicting Future Trend of Customers in Banking Sectors in India using Mathematical Modeling

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Abstract---A mathematical model for determining customers' population in various banking sectors in India has been developed with the aid of compartmental diagrams. For deriving this model, the concept of rate of change has been used. The model emerges in the form of differential equations which have been solved with the help of linear algebra technique. A solution of these differential equations has been shown graphically using the mathematical software Mathematica, so that the solution can be visualized and understood by anyone having no higher mathematical background. Numerical discussion has been carried out for hypothetical data taken as per observed trend of customers in various banking sectors in the current scenario. The present study shows that private and foreign banks dominate the public sector and cooperative banks in context of number of customers dealing. Private and foreign banks show continuous progress in attracting customers whereas public sector and cooperative banks take half of the time span of 100 years to show positive performance and to create confidence of the customers. The scope of future research has also been proposed in terms of finding the possible non-zero solutions of homogenous system of linear equations resulting in evaluation of equilibrium points lying in number of customers of various banking sectors.

Keywords--- compartmental diagram, per-capita rates, equilibrium points, trajectories, eigen values, eigenvectors.

Introduction

Banking industry is the backbone in the skeleton of financial security modes of any human civilized population. Four types of banking sectors (public sector banks, private banks, foreign banks and co-operative banks) can be seen operating in banking industry of various developed or developing countries. The stakeholders taking services of these banking sectors often encounter with a challenge in selection of the banking sector suitable for their present situation and fulfilling their future needs. Liking or disliking of customers, the stakeholders, for any banking sector get affected by many factors. In the present competitive scenario, banking sectors offer lucrative schemes to attract customers from other sectors. Apart from this aspect, multiple reasons play role in the movement of customers from one sector to other, some of which are enlisted below:

- Private banks are more techno-savvy in comparison to public sector banks. The average tech-savvy Indians are gradually shifting their preference towards private banks.

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- Better and quick services in private and foreign banks.
- Main reasons for a borrower to shift are the faster sanctions and disbursement of consumer loans, Lower interest rates and processing charges.
- Private banks are most focused on retail segment whereas public sector banks are more focused on Corporate.
- A higher rate of interest on saving bank account (i.e., 6%) by certain private banks like Kotak Mahindra Bank and Yes Bank. Otherwise, the general rate of interest on saving bank offered by a Bank is 4%.
- A perception in the eyes of general public that private banks are more efficient as compared to Public Sector Banks.
- A perception in the eyes of middle-age and senior citizens public that public sector banks are safer as compared to private Banks.
- Convenient location of banks also affects the choice of a bank.
- Public in the rural and remote areas prefer the cooperative banks.
- Foreign customers choose foreign and private banks.
- Care for sentimental aspects also helps in attracting customers.

Banking industry always needs future aspects from different angles for making their strategies for their development. One major factor of growth of any banking sector is the number of customers availing their services. In the banking sector in India, there is a healthy competition between the four major sectors of banks: public sector banks (PSB), private banks (PB), foreign banks (FB), co-operative banks (CB). An efficient and logical method to deal with any kind of real-world problems is the mathematical modeling. There is hardly any field left which was not grabbed and dominated by mathematical modeling, be it be physical sciences, social sciences, medical science, agricultural science, commerce and management to name a few (Chambers, 2000; Peppas, 2013; Luce, 1995; Clarke & Skiba, 2013; Melnik, 2015; Mikhailov et al., 2015; France, 1988; Ruhanian & Movagharnejad, 2016; Cortés et al., 2010; Feng & Huang, 2016; Garg, 2007; Garg, 2009; Rao et al., 2016; Ledder, 2008). In banking industry also, many researchers applied mathematical techniques to develop mathematical models to deal with different types of objectives. One of the major concerns in banking system is to manage the asset liability in order to gain more profit by minimizing any kind of risk. Many researchers dealt with this problem using mathematical models and used the techniques of Goal Programming (GP) along with Analytical Hierarchy Process (AHP) (Kruger, 2011; Sedzro et al., 2012; Kamil et al., 2013; Halim et al., 2015). In 2013, Smeureanu and coworkers (2013) used two techniques of machine learning to study the segmentation of customers in private banks required to increase the profit of banking business. Selyutin and Rudenko (2013) succeeded in performing the analysis, management and learning of a bank using mathematical modeling. Further, studies on relative efficiency of banks using optimization technique, a nonparametric mathematical analysis can be found in literature (Evanroff & Israilevich, 1991; Altunbas et al., 2000; Alam, 2001; Almumani, 2003; Aikaeli, 2008; Zeitun & Benjelloun, 2013). In the present paper, authors proposed a model to predict the future trend of customers in these four banking sectors. The mathematical techniques used in the present study include differential equations and linear algebra. Some of the books and research work for these mathematical techniques available in literature are given for reference (Banks, 1994; Barnes & Fulford, 2009; Borelli & Coleman, 1996; Braun, 1979; Gani, 1980; Hurewicz, 1990; Lay, 1994; Mesterton-Gibbons, 1989).

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Model Assumptions

1. The customers in the banking sectors are large, so that the random differences between individuals can be neglected.
2. The only banking sectors affecting the system are: public sector banks (PSB), private banks (PB), foreign banks (FB) and co-operative banks (CB).
3. In case of absence of any three banking sectors, the remaining fourth banking sector will grow exponentially.
4. There is continuous growth in all the four banking sectors instead of discrete growth over time.

The interaction between these banking sectors can be understood by Figure 1. All the four banking sectors compete for the same customers. Also, there is a possibility that customers of one sector may shift to other sectors. So, gain of one sector in terms of customers will be the loss of other sector. For each sector, there will be three competitors as depicted in Figure 1. The incoming and outgoing of customers in a sector will be the input and output respectively for that sector. The compartmental diagram (input-output diagram for a system over passage of time) for a banking sector is given in Figure 2.

Suppose that the total population size in India at any time t be N(t), out of which M(t) has the association with banks, and the remaining population (RP), i.e., R(t) = N(t) − M(t), has no dealing with banks. The remaining population R(t) also has the possibility to enter any banking

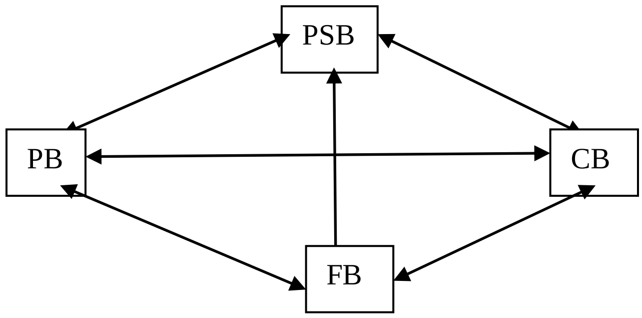

Figure 1. Interaction between the banking sectors in India

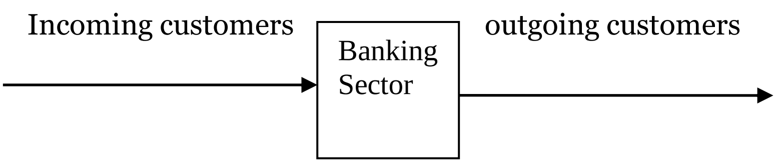

Figure 2. Compartmental diagram for a banking sector

sector depending upon their convenience, comfort and interest in a banking sector. Let the number of customers in PSB, PB, FB and CB be denoted by A(t), B(t), C(t) and D(t) respectively. The words equation to calculate the growth rate in terms of customers of a banking sector X is given as,

{rate of change of customers in X}={rate of customers entry in X}-{rate of customers exit from X} (1)

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Formulating the Differential Equations

Differential Equation for PSB

The input diagram for PSB is as shown in Figure 3. The assumptions made to determine the inputs for PSB are as under.

- A birth in the family of customers of PSB will provide a new customer to PSB.
- Assume that some attractive offer is given by PSB. Then from the remaining public R(t), a new customer may enter PSB.
- Assume those competitions between banking sectors in which PSB is one of the competitors and also the winner, i.e., PSB gains the customers from other banking sectors.

Based upon these assumptions, various rates of inputs for PSB are given as,

{rate of customers entry due to birth in the family of customers already present in PSB} ∞ A

i.e.,

{rate of customers entry due to birth in the family of customers already present in PSB} = a_1 A,

where a_1 is a positive constant of proportionality.

{rate of customers entry in PSB from remaining population (RP)} ∞ R

i.e.,

{rate of customers entry in PSB from remaining population (RP)} = a_2 R,

where a_2 is a positive constant of proportionality.

{rate of gain of customers in PSB from PB} ∞ B

i.e.,

{rate of gain of customers in PSB from PB} = a_3 B,

where a_3 is a positive constant of proportionality.

{rate of gain of customers in PSB from FB} ∞ C

i.e.,

{rate of gain of customers in PSB from FB} = a_4 C,

where a_4 is a positive constant of proportionality.

{rate of gain of customers in PSB from CB} ∞ D

i.e.,

{rate of gain of customers in PSB from CB} = a_5 D,

where a_5 is a positive constant of proportionality.

Here, the interpretation for the constants may be taken as,

a_1: per-capita birth rate of customers in PSB
a_2: per-capita entry rate of customers in PSB from remaining public (RP)
a_3: per-capita gain rate of customers in PSB from PB
a_4: per-capita gain rate of customers in PSB from FB
a_5: per-capita gain rate of customers in PSB from CB

Therefore,

{rate of gain of customers in PSB} = a_1 A + a_2 R + a_3 B + a_4 C + a_5 D

Now, the output diagram for PSB is as shown in Figure 4. The assumptions made to determine the outputs for PSB are as under.

- A death in the family of customers of PSB will be loss of a customer to PSB.

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- Some customers may exit from PSB after getting their requirements fulfilled or due to some other reason. Assume that those customers do not enter in any other banking sector.
- Assume those competitions between banking sectors in which PSB is one of the competitors and also the loser, i.e., PSB loses the customers who switch over to other banking sectors.

Based upon these assumptions, various rates of outputs for PSB are given as,

{rate of loss of customersin PSB due to death} ∞ A

i.e.,

{rate of loss of customersin PSB due to death} = p_1 A,

where p_1 is a positive constant of proportionality.

{rate of exit of customers from PSB but not entering in any other banking sector} ∞ A

i.e.,

{rate of exit of customers from PSB but not entering in any other banking sector} = p_2 A,

where p_2 is a positive constant of proportionality.

{rate of loss of customers in PSB due to their shifting to PB, FB and CB} ∞ A

i.e.,

{rate of loss of customers in PSB due to their shifting to PB, FB and CB} = $(p_3 + p_4 + p_5)$ A,

where p_3 , p_4 , p_5 are positive constants of proportionality.

Here, the interpretation for the constants may be taken as,

p_1: per-capita death rate of customers in PSB

p_2: per-capita exit rate of customers from PSB but not entering in any other banking sector

p_3: per-capita exit rate of customers from PSB and entering to PB

p_4: per-capita exit rate of customers from PSB and entering to FB

p_5: per-capita exit rate of customers from PSB and entering to CB

Therefore,

{rate of loss of customers in PSB} = $(p_1 + p_2 + p_3 + p_4 + p_5)$ A

Thus,

{rate of change of customers in PSB}

= {rate of gain of customers in PSB} - {rate of loss of customers in PSB}

i.e.,

$$\frac{dA}{dt} = (a_1 A + a_2 R + a_3 B + a_4 C + a_5 D) - (p_1 + p_2 + p_3 + p_4 + p_5) A$$

i.e.,

$$\frac{dA}{dt} = (a_1 - p_1 - p_2 - p_3 - p_4 - p_5) A + a_2 R + a_3 B + a_4 C + a_5 D \qquad (2)$$

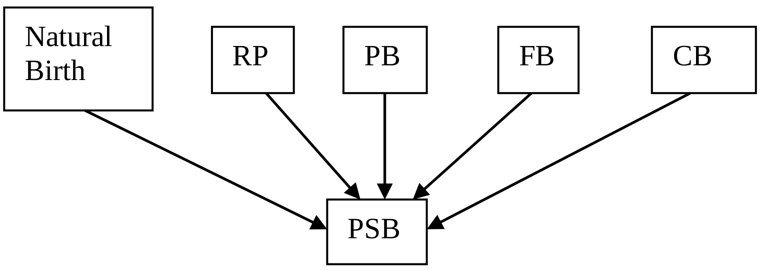

Figure 3. Input diagram for PSB

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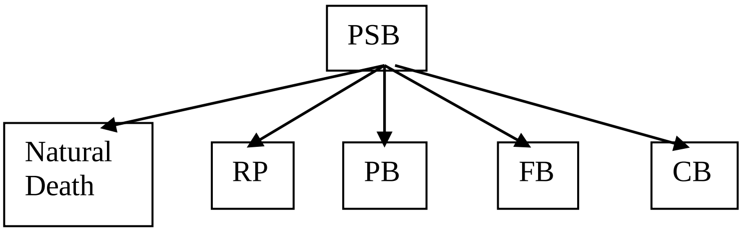

Figure 4. Output diagram for PSB

Differential Equation for PB

The input and output diagram for PB are as shown in Figure 5 and Figure 6 respectively.
As calculated for PSB, the rate of change of customers in PB is given as,

$$\frac{dB}{dt} = (b_1 B + b_2 R + p_3 A + b_3 C + b_4 D) - (q_1 + q_2 + a_3 + q_3 + q_4) B$$

i.e.,

$$\frac{dB}{dt} = p_3 A + (b_1 - q_1 - q_2 - a_3 - q_3 - q_4) B + b_3 C + b_4 D + b_2 R \qquad (3)$$

Here, the interpretation for the constants may be taken as,

b_1: per-capita birth rate of customers in PB
b_2: per-capita entry rate of customers in PB who have no public dealing (RP)
p_3: per-capita gain rate of customers in PB from PSB
b_3: per-capita gain rate of customers in PB from FB
b_4: per-capita gain rate of customers in PB from CB
q_1: per-capita death rate of customers in PB
q_2: per-capita exit rate of customers from PB but not entering in any other banking sector
a_3: per-capita loss rate of customers from PB and entering to PSB
q_3: per-capita loss rate of customers from PB and entering to FB
q_4: per-capita loss rate of customers from PB and entering to CB

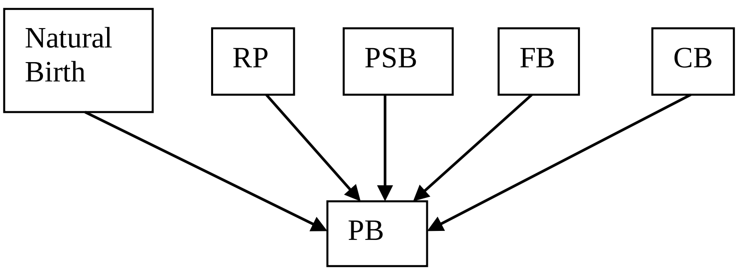

Figure 5. Input diagram for PB

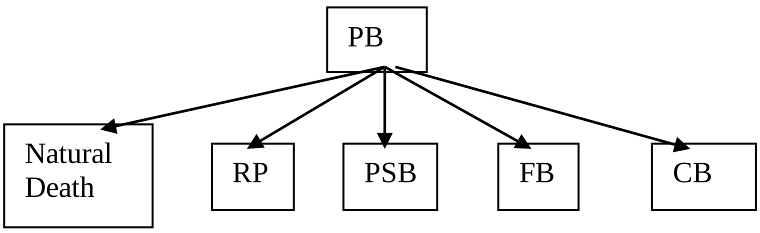

Figure 6. Output diagram for PB

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Differential Equation for FB

The input and output diagram for FB are as shown in Figure 7 and Figure 8 respectively.
As calculated for PSB, the rate of change of customers in FB is given as,

$$\frac{dC}{dt} = (c_1 C + c_2 R + p_4 A + q_3 B + c_3 D) - (r_1 + r_2 + a_4 + b_3 + r_3) C$$

i.e.,

$$\frac{dC}{dt} = p_4 A + q_3 B + (c_1 - r_1 - r_2 - a_4 - b_3 - r_3) C + c_3 D + c_2 R \quad (4)$$

Here, the interpretation for the constants may be taken as,

c_1: per-capita birth rate of customers in FB
c_2: per-capita entry rate of customers in FB who have no public dealing (RP)
p_4: per-capita gain rate of customers in FB from PSB
q_3: per-capita gain rate of customers in FB from PB
c_3: per-capita gain rate of customers in FB from CB
r_1: per-capita death rate of customers in FB
r_2: per-capita exit rate of customers from FB but not entering in any other banking sector
a_4: per-capita loss rate of customers from FB and entering to PSB
b_3: per-capita loss rate of customers from FB and entering to PB
r_3: per-capita loss rate of customers from FB and entering to CB

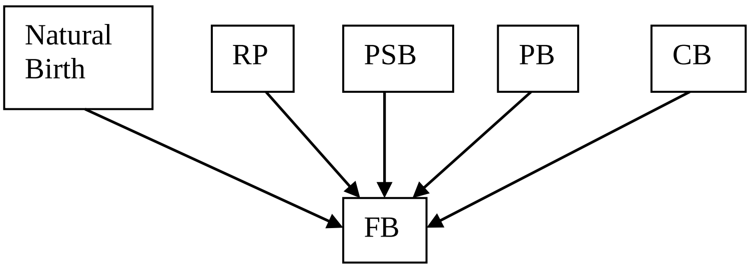

Figure 7. Input diagram for FB

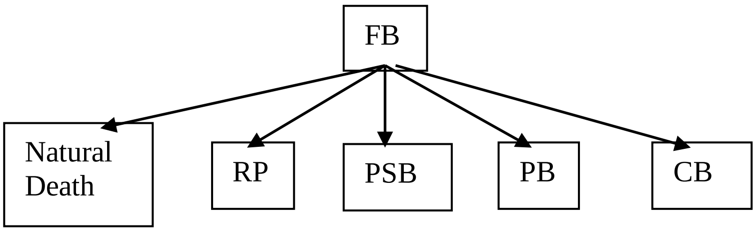

Figure 8. Output diagram for FB

Differential Equation for CB

The input and output diagram for FB are as shown in Figure 9 and Figure 10 respectively.
As calculated for PSB, the rate of change of customers in CB is given as,

$$\frac{dD}{dt} = (d_1 D + d_2 R + p_5 A + q_4 B + r_3 C) - (s_1 + s_2 + a_5 + b_4 + c_3) D$$

i.e.,

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$$\frac{dD}{dt} = p_5\ A + q_4\ B + r_3\ C + (d_1 - s_1 - s_2 - a_5 - b_4 - c_3)\ D + d_2\ R \qquad (5)$$

Here, the interpretation for the constants may be taken as,
d_1: per-capita birth rate of customers in CB
d_2: per-capita gain rate of customers in CB who have no public dealing (RP)
p_5: per-capita gain rate of customers in CB from PSB
q_4: per-capita gain rate of customers in CB from PB
r_3: per-capita gain rate of customers in CB from FB
s_1: per-capita death rate of customers in CB
s_2: per-capita loss rate of customers from CB but not entering in any other banking sector
a_5: per-capita loss rate of customers from CB and entering to PSB
b_4: per-capita loss rate of customers from CB and entering to PB
c_3: per-capita loss rate of customers from CB and entering to FB

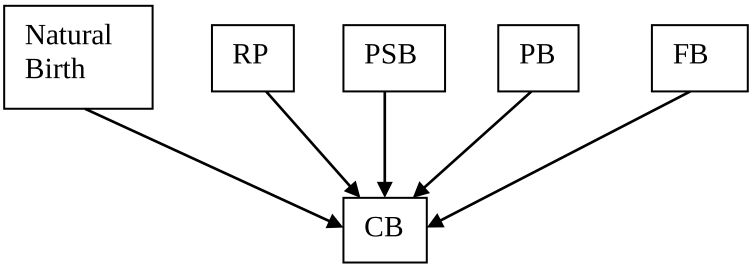

Figure 9. Input diagram for CB

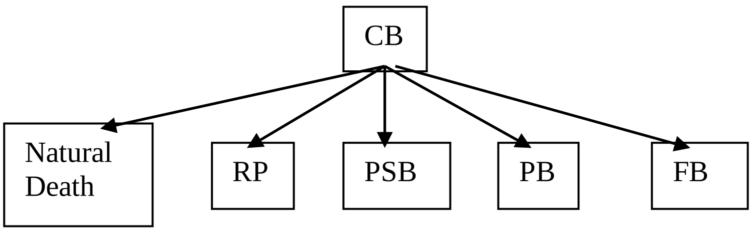

Figure 10. Output diagram for CB

Differential Equation for RP

The input and output diagram for FB are as shown in Figure 11 and Figure 12 respectively. As calculated for PSB, the rate of change of remaining population (RP) is given as,

$$\frac{dR}{dt} = (e_1\ R + p_2\ A + q_2\ B + r_2\ C + s_2\ D) - (t_1 + a_2 + b_2 + c_2 + d_2)\ R$$

i.e.,

$$\frac{dR}{dt} = p_2\ A + q_2\ B + r_2\ C + s_2\ D + (e_1 - t_1 - a_2 - b_2 - c_2 - d_2)\ R \qquad (6)$$

where e_1 and t_1 are per-capita birth rate and death rate of remaining public (RB) respectively. Thus, the mathematical model obtained to find the future trend of customers in banking sector is,

$$\frac{dA}{dt} = (a_1 - p_1 - p_2 - p_3 - p_4 - p_5)\ A + a_3\ B + a_4\ C + a_5\ D + a_2\ R$$

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$$\frac{dB}{dt} = p_3\ A + (\ b_1 - q_1\ - q_2 - a_3\ - q_3 - q_4)\ B\ +\ b_3\ C + b_4\ D + b_2\ R$$
$$\frac{dC}{dt} = p_4\ A + q_3\ B + (\ c_1 - r_1\ - r_2 - a_4\ - b_3 - r_3)\ C + c_3\ D + c_2\ R$$
$$\frac{dD}{dt} = p_5\ A + q_4\ B + r_3\ C + (\ d_1 - s_1\ - s_2 - a_5\ - b_4 - c_3)\ D + d_2\ R$$
$$\frac{dR}{dt} = p_2\ A + q_2\ B + r_2\ C + s_2\ D + (e_1 - t_1\ - a_2 - b_2\ - c_2 - d_2)\ R \qquad (7)$$

These are coupled five linear first-order differential equations in five unknown variables and thirty parameters. By giving different values to these parameters, a wide scope will be there to explore a variety of future trends of customers in banking sectors.

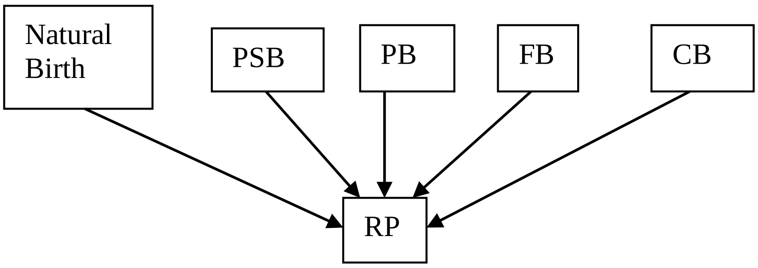

Figure 11. Input diagram for RP

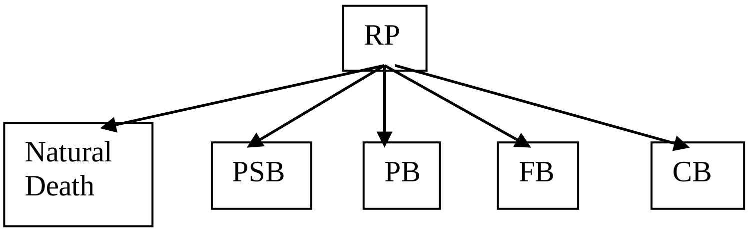

Figure 12. Output diagram for RP

Equilibrium Points

Equilibrium points means that there will be no rate of change in A, B, C, D and R. Thus, the equations used to calculate the equilibrium points will be given as,

$$\frac{dA}{dt} = 0\ ,\ \frac{dB}{dt} = 0\ ,\ \frac{dC}{dt} = 0\ ,\ \frac{dD}{dt} = 0\ ,\ \frac{dR}{dt} = 0\ . \qquad (8)$$

Making use of equations (7) in equations (8) provide the equations,

$$(\ a_1 - p_1 - p_2 - p_3 - p_4 - p_5)\ A +\ a_3\ B + a_4\ C + a_5\ D + a_2\ R = 0$$
$$p_3\ A + (\ b_1 - q_1\ - q_2 - a_3\ - q_3 - q_4)\ B\ +\ b_3\ C + b_4\ D + b_2\ R = 0$$
$$p_4\ A + q_3\ B + (\ c_1 - r_1\ - r_2 - a_4\ - b_3 - r_3)\ C + c_3\ D + c_2\ R = 0$$
$$p_5\ A + q_4\ B + r_3\ C + (\ d_1 - s_1\ - s_2 - a_5\ - b_4 - c_3)\ D + d_2\ R = 0$$
$$p_2\ A + q_2\ B + r_2\ C + s_2\ D + (e_1 - t_1\ - a_2 - b_2\ - c_2 - d_2)\ R = 0 \qquad (9)$$

which is a system of linear homogeneous equations in five variables A, B, C, D and R. This system has a non-zero solution if there is linear dependency among any of the five vectors: $(a_1-p_1-p_2-p_3-p_4-p_5, a_3, a_4, a_5, a_2)$, $(p_3, b_1-q_1-q_2-a_3-q_3-q_4, b_3, b_4, b_2)$, $(p_4, q_3, c_1-r_1-r_2-a_4-b_3-r_3, c_3, c_2)$, $(p_5, q_4, r_3, d_1-s_1-s_2-a_5-b_4-c_3, d_2)$ and $(p_2, q_2, r_2, s_2, e_1-t_1-a_2-b_2-c_2-d_2)$. But practically this

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situation is very rare. So, this possibility may be taken as one of the objectives of further study. The other possible solution of system (9) is the zero solution which gives an equilibrium point of the banking system as,

$(A_e, B_e, C_e, D_e, R_e) = (0, 0, 0, 0, 0)$

Trajectories of Banking Sectors

Trajectories of various banking sectors can be determined by the solution of system (7) which will be obtained with the help of matrices as explained in (Barnes & Fulford, 2009). In matrix notation, system (7) can be written as,

$$n' = P\,n \tag{10}$$

where

$$n' = \begin{pmatrix} \frac{dA}{dt} & \frac{dB}{dt} & \frac{dC}{dt} & \frac{dD}{dt} & \frac{dR}{dt} \end{pmatrix}^T$$

$$P = \begin{pmatrix} a_1 - p_1 - p_2 - p_3 - p_4 - p_5 & a_3 & a_4 & a_5 & a_2 \\ p_3 & b_1 - q_1 - q_2 - a_3 - q_3 - q_4 & b_3 & b_4 & b_2 \\ p_4 & q_3 & c_1 - r_1 - r_2 - a_1 - b_3 - r_3 & c_3 & c_2 \\ p_5 & q_4 & r_3 & d_1 - s_1 - s_2 - a_5 - b_4 - c_3 & d_2 \\ p_2 & q_2 & r_2 & s_2 & e_1 - t_1 - a_2 - b_2 - c_2 - d_2 \end{pmatrix}$$

$$n = \begin{pmatrix} A & B & C & D & R \end{pmatrix}^T$$

In the above notations, the superscript T represents the transpose of a matrix.

The eigenvalues λ of matrix P are given by the equation,

$$\det(P - \lambda I) = 0, \tag{11}$$

where det represents the determinant of a matrix, I is the identity matrix of order five, λ is a scalar. Equation (11) provides an algebraic equation of degree five in λ, and hence gives five values for λ on solution known as eigenvalues for the matrix P. Take these values as λ_i , $1 \le i \le 5$.

Eigenvectors X (≠ **0** vector) associated with λ for the matrix P are obtained by the solution of the equations,

$$(P - \lambda I)\,X = 0 \tag{12}$$

If u, v, w, x and y represent the associated eigenvectors for eigenvalues λ_1, λ_2, λ_3, λ_4, and λ_5 respectively, where

$$u = (u_1\ u_2\ u_3\ u_4\ u_5)^T,\ v = (v_1\ v_2\ v_3\ v_4\ v_5)^T,\ w = (w_1\ w_2\ w_3\ w_4\ w_5)^T,$$
$$x = (x_1\ x_2\ x_3\ x_4\ x_5)^T,\ y = (y_1\ y_2\ y_3\ y_4\ y_5)^T \tag{13}$$

Then,

$$P\,u = \lambda_1 u,\ P\,v = \lambda_2 v,\ P\,w = \lambda_3 w,\ P\,x = \lambda_4 x,\ P\,y = \lambda_5 y. \tag{14}$$

Now, define a matrix,

$$U = (u\ v\ w\ x\ y) \tag{15}$$

Thus, the matrix U is,

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$$U = \begin{pmatrix} u_1 & v_1 & w_1 & x_1 & y_1 \\ u_2 & v_2 & w_2 & x_2 & y_2 \\ u_3 & v_3 & w_3 & x_3 & y_3 \\ u_4 & v_4 & w_4 & x_4 & y_4 \\ u_5 & v_5 & w_5 & x_5 & y_5 \end{pmatrix} \quad (16)$$

Now,
P U = P (u v w x y) = (Pu Pv Pw PxPy)
Making use of relations (14),
P U = (λ_1u λ_2v λ_3w λ_4x λ_5y),
which can further be written as,
P U = (u v w x y) diag (λ_1 λ_2 λ_3 λ_4 λ_5)
where second matrix on right side is a diagonal matrix.
In symbolic notation,
P U = U Q, (17)
where,
Q = diag (λ_1 λ_2 λ_3 λ_4 λ_5)
In the case, where the thirty parameters in system (7) have those values for which matrix U is invertible, equation (17) provides,
U^{-1} P U = Q (18)
Assume that vector n can be expressed as a linear combination of eigenvectors given as,
n = α_1 u + α_2 v + α_3 w + α_4 x + α_5 y (19)
where α_i , $1 \le i \le 5$ are the scalers.
Define a vector α as, α = $(\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4\ \alpha_5)^T$. Then the vector n becomes,
n = U α (20)
The matrix n is comprised of A, B, C, D, E, which are functions of time t. But the matrix U is comprised of the eigenvectors u, v, w, x, y, which are not the functions of time t since the matrix P, of which u, v, w, x, y are the eigenvectors, does not depend upon time t. The fact that left side of equation (20) is a function of time t, reveals that the vector α on right side of equation (20) should be a function of time t. Thus, differentiation of equation (20) with respect to time t gives,
n' = U α' (21)
Making use of value n from relation (20) in relation (10) yields,
n' = P U α (22)
Comparison of values of n' from (21) and (22) gives,
U α' = P U α
Pre-multiplying both sides by U^{-1} gives,
α' = U^{-1} P U α
which provides with the aid of (18),
α' = Q α
i.e.,
$(\alpha_1'\ \alpha_2'\ \alpha_3'\ \alpha_4'\ \alpha_5')^T = (\lambda_1\alpha_1\ \ \lambda_2\alpha_2\ \ \lambda_3\alpha_3\ \ \lambda_4\alpha_4\ \ \lambda_5\alpha_5)^T$
Comparing the elements both sides,
$\alpha_i' = \lambda_i \alpha_i$, where i can take values 1, 2, 3, 4, 5. (23)
Solution of first order differential equations (23) provides,
$\alpha_i = k_i e^{\lambda_i t}$, (24)
where k_i is the arbitrary constant.
Making use of (24) in (19),
n = $k_1 e^{\lambda_1 t}$ u + $k_2 e^{\lambda_2 t}$ v + $k_3 e^{\lambda_3 t}$ w + $k_4 e^{\lambda_4 t}$ x + $k_5 e^{\lambda_5 t}$ y (25)
Define the new vectors,
u' = k_1 u, v' = k_2 v, w' = k_3 w, x' = k_4 x, y' = k_5y, (26)

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where $u' = (u_1' \; u_2' \; u_3' \; u_4' \; u_5')^T$, $v' = (v_1' \; v_2' \; v_3' \; v_4' \; v_5')^T$, $w' = (w_1' \; w_2' \; w_3' \; w_4' \; w_5')^T$,
$x' = (x_1' \; x_2' \; x_3' \; x_4' \; x_5')^T$, $y' = (y_1' \; y_2' \; y_3' \; y_4' \; y_5')^T$
Here, u', v', w', x' and y' are the eigenvectors associated with λ_1, λ_2, λ_3, λ_4, λ_5 respectively, as scalar multiple of an eigenvector is again an eigenvector associated with the same eigenvalue.
With the aid of (26), the value of n in (25) becomes,

$$n = e^{\lambda_1 t} u' + e^{\lambda_2 t} v' + e^{\lambda_3 t} w' + e^{\lambda_4 t} x' + e^{\lambda_5 t} y' \quad (27)$$

Comparing the components on both sides, the number of customers in various banking sectors is obtained as,

$$A = e^{\lambda_1 t} u_1' + e^{\lambda_2 t} v_1' + e^{\lambda_3 t} w_1' + e^{\lambda_4 t} x_1' + e^{\lambda_5 t} y_1'$$
$$B = e^{\lambda_1 t} u_2' + e^{\lambda_2 t} v_2' + e^{\lambda_3 t} w_2' + e^{\lambda_4 t} x_2' + e^{\lambda_5 t} y_2'$$
$$C = e^{\lambda_1 t} u_3' + e^{\lambda_2 t} v_3' + e^{\lambda_3 t} w_3' + e^{\lambda_4 t} x_3' + e^{\lambda_5 t} y_3'$$
$$D = e^{\lambda_1 t} u_4' + e^{\lambda_2 t} v_4' + e^{\lambda_3 t} w_4' + e^{\lambda_4 t} x_4' + e^{\lambda_5 t} y_4'$$
$$R = e^{\lambda_1 t} u_5' + e^{\lambda_2 t} v_5' + e^{\lambda_3 t} w_5' + e^{\lambda_4 t} x_5' + e^{\lambda_5 t} y_5' \quad (28)$$

Thus, by evaluating eigenvalues and eigenvectors of matrix P, trajectories of population of customers in various banking sectors can be estimated at any time.

Numerical Discussion

To support the mathematical model obtained above, consider the hypothetical values for the parameters present in differential equations as:
$a_1 = 0.001$, $a_2 = 0.3$, $a_3 = 0.01$, $a_4 = 0.01$, $a_5 = 0.1$;
$p_1 = 0.0005$, $p_2 = 0.001$, $p_3 = 0.1$, $p_4 = 0.01$, $p_5 = 0.01$;
$b_1 = 0.01$, $b_2 = 0.0001$, $b_3 = 0.01$, $b_4 = 0.01$, $q_1 = 0.001$, $q_2 = 0.001$, $q_3 = 0.001$, $q_4 = 0.0001$;
$c_1 = 0.01$, $c_2 = 0.00005$, $c_3 = 0.001$, $r_1 = 0.001$, $r_2 = 0.001$, $r_3 = 0.00005$;
$d_1 = 0.002$, $d_2 = 0.5$, $s_1 = 0.0005$, $s_2 = 0.002$; $e_1 = 0.03$, $t_1 = 0.01$; (29)
Assume the initial customers population as:
$A(t=0) = 1 \times 10^8$, $B(t=0) = 3 \times 10^7$, $C(t=0) = 5 \times 10^6$, $D(t=0) = 5 \times 10^7$; (30)
Therefore, the initial remaining population is,
$R(t=0) = 1095 \times 10^6$
With the assumed values in (29), the matrix P takes the values as,
P = [-0.1205, 0.01, 0.01, 0.1, 0.3; 0.1, -0.0031, 0.01, 0.01, 0.0001; 0.01, 0.001, -0.00305, 0.001, 0.00005; 0.01, 0.0001, 0.00005, -0.1115, 0.5; 0.001, 0.001, 0.001, 0.002, -0.78015]
Eigenvalues obtained for this matrix P using the software Mathematica are as,
$\lambda_1 = -0.781915$, $\lambda_2 = -0.151911$, $\lambda_3 = -0.0885607$, $\lambda_4 = 0.00938365$, $\lambda_5 = -0.00529615$
Eigenvectors associated with these eigenvalues using Mathematica are calculated as,
$u' = k_1 (-0.265106, 0.0412507, 0.00403727, -0.573382, 0.774102)$
$v' = k_2 (0.820093, -0.534394, -0.0501685, -0.198377, -0.000256625)$
$w' = k_3 (-0.611182, 0.739524, 0.0660333, -0.274227, -0.000511979)$
$x' = k_4 (0.104395, 0.98076, 0.164153, 0.0162301, 0.00162345)$
$y' = k_5 (-0.0338493, -0.847019, 0.530451, -0.00593623, -0.000467558)$
Thus, the customers' population in various banking sectors becomes,

$$A = -0.265106\, e^{-0.781915t} k_1 + 0.820093\, e^{-0.151911t} k_2 - 0.611182\, e^{-0.088561t} k_3 + 0.104395\, e^{0.009384t} k_4 - 0.033849\, e^{-0.005296t} k_5$$
$$B = 0.041251\, e^{-0.781915t} k_1 - 0.534394\, e^{-0.151911t} k_2 + 0.739524\, e^{-0.088561t} k_3 + 0.98076\, e^{0.009384t} k_4 - 0.847019\, e^{-0.005296t} k_5$$
$$C = 0.004037 e^{-0.781915t} k_1 - 0.050168 e^{-0.151911t} k_2 + 0.066033 e^{-0.088561t} k_3 + 0.164153 e^{0.009384t} k_4 + 0.530451 e^{-0.005296t} k_5$$
$$D = -0.573382 e^{-0.781915t} k_1 - 0.198377 e^{-0.151911t} k_2 - 0.274227\, e^{-0.088561t} k_3 + 0.016230 e^{0.009384t} k_4 - 0.005936 e^{-0.005296t} k_5$$
$$R = 0.774102 e^{-0.781915t} k_1 - 0.000257 e^{-0.151911t} k_2 - 0.000512\, e^{-0.088561t} k_3 + 0.001623 e^{0.009384t} k_4 - 0.000468 e^{-0.005296t} k_5$$

Using the assumed initial customers' population in (30), the reduced non-homogeneous system of five equations provides the solution as,

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$k_1 = 1.41103\times10^8$,$k_2 = -1.11029\times10^9$, $k_3 = -2.1119\times10^9$, $k_4 = 8.63228\times10^8$, $k_5 = -1.10555\times10^8$

Using these values of k_i's (I = 1, 2, 3, 4, 5), the customers' population in various banking sectors are given by the equations,

$$A = - 3.74073\times10^7e^{-0.781915t} - 9.10541\times10^8e^{-0.151911t}+1.29076\times10^9e^{-0.088561t} +9.01167\times10^7 e^{0.009384t}-3.74218\times10^6e^{-0.005296t}$$

$$B = 5.82064\times10^6e^{-0.781915t} +5.93332\times10^8e^{-0.151911t} -1.5618\times10^9e^{-0.088561t} +8.46619\times10^8e^{0.009384t} +9.36422\times10^7e^{-0.005296t}$$

$$C = 569633e^{-0.781915t} +5.5701\times10^7e^{-0.151911t} -1.39455\times10^8e^{-0.088561t} +1.41701\times10^8e^{0.009384t} -5.8644\times10^7e^{-0.005296t}$$

$$D = -8.09059\times10^7e^{-0.781915t} +2.20256\times10^8e^{-0.151911t} +5.7914\times10^8e^{-0.088561t} +1.40102\times10^7e^{0.009384t} +656254e^{-0.005296t}$$

$$R = 1.09228\times10^8e^{-0.781915t} +285345e^{-0.151911t} +1.08129\times10^6e^{-0.088561t} +1.40102\times10^6e^{0.009384t} +51739.7e^{-0.005296t}$$

These equations can be used to determine customers' population in various banking sectors. Using these equations, customers' population at various times is given in Table 1. It can be seen that for the assumed data, customers' population in PSB goes on decreasing in the first fifty years, but after that it shows increasing trend. In the similar fashion, CB has negative phase in the first sixty years and after that positive phase in the next forty years. The same trend can be seen in RP, which decreases first in the first thirty years and then increases in the next seventy years. In contrast, PB and FB enjoys only the increasing phase in the first hundred years.

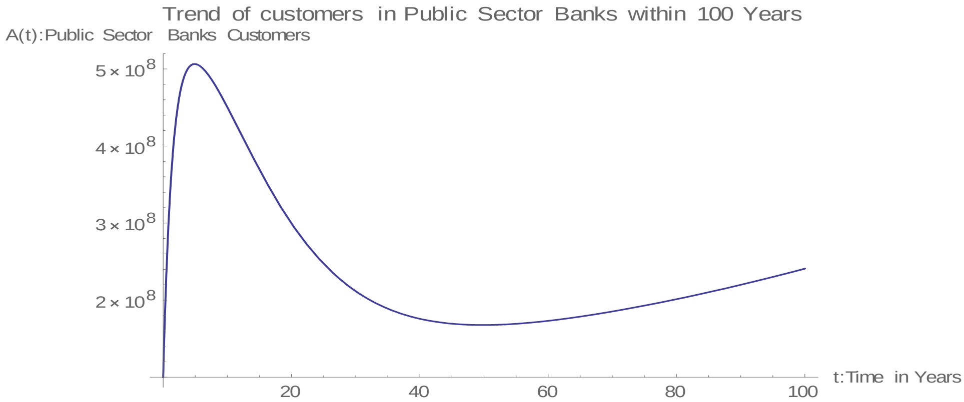

Figure 13. Trend of customers in PSB within 100 years

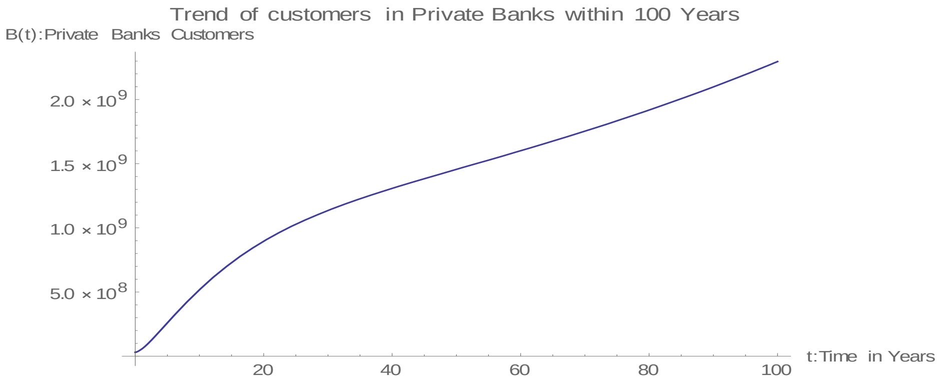

Figure 14. Trend of customers in PB within 100 years

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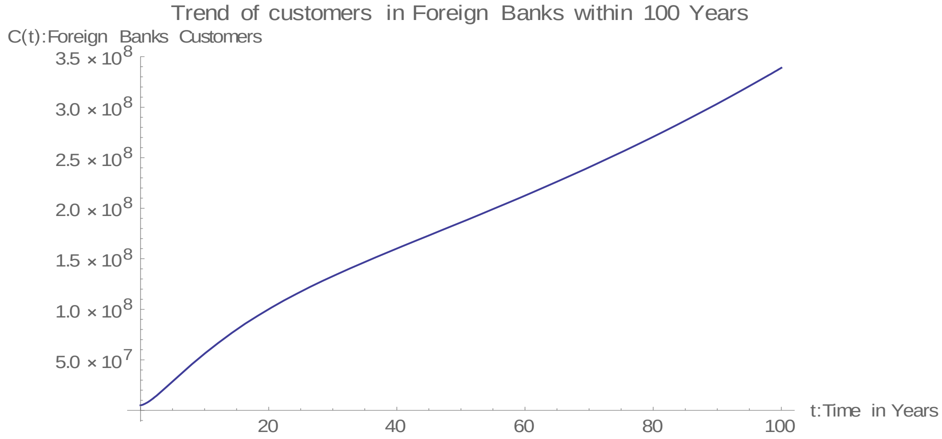

Figure 15. Trend of customers in FB within 100 years

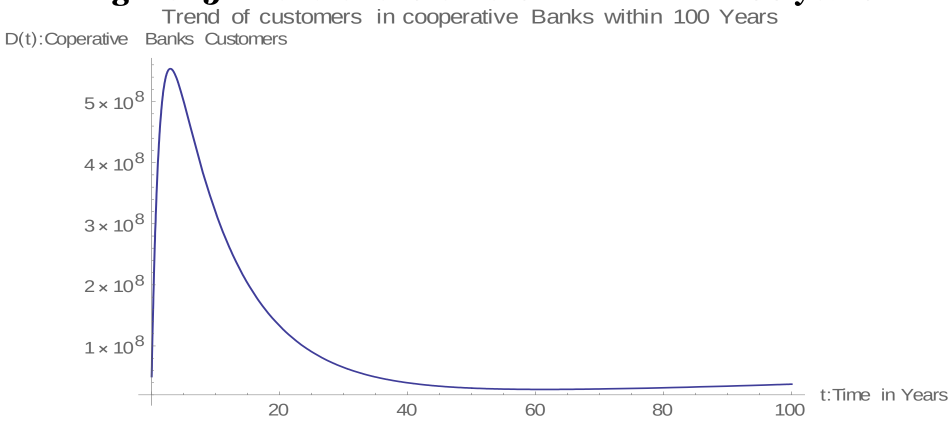

Figure 16. Trend of customers in CB within 100 years

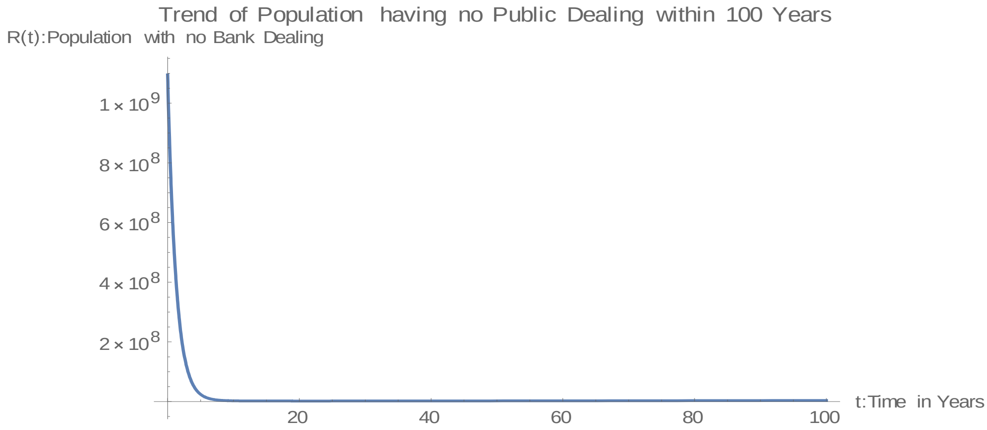

Figure 17. Trend of customers in RP within 100 years

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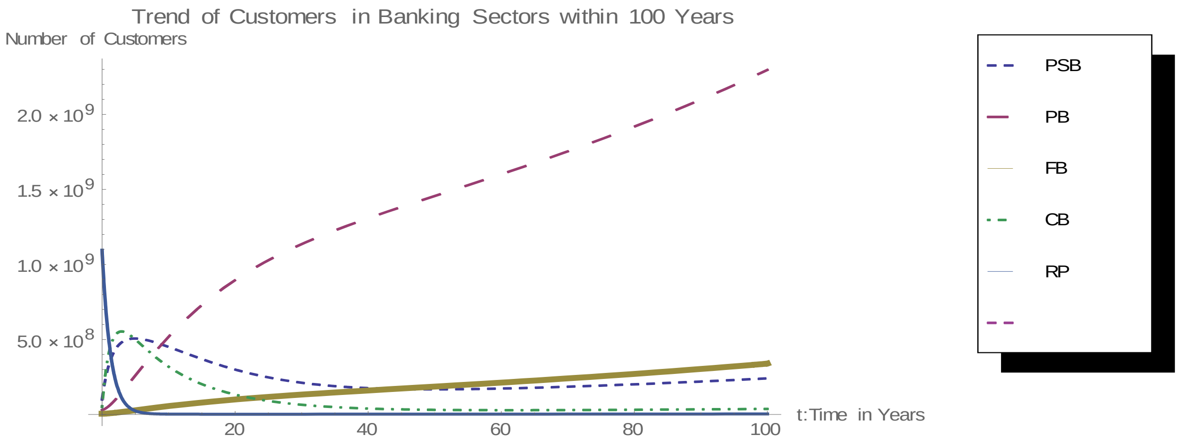

Figure 18. Trend of customers in various banking sectors within 100 years

Table 1. Customers' population in various banking sectors at various times

t (Time in years)	A (Customers' population in PSB)	B (Customers' population in PB)	C (Customers' population in FB)	D (Customers' population in CB)	R (Customers' population in RP)
10	4.32573×10^8	5.04427×10^8	5.46967×10^7	3.03067×10^8	2.14029×10^6
20	2.85756×10^8	8.68363×10^8	9.71491×10^7	1.26574×10^8	1.93444×10^6
30	2.02055×10^8	1.09841×10^9	1.28544×10^8	6.20743×10^7	1.97956×10^6
40	1.68588×10^8	1.26419×10^9	1.54891×10^8	3.81906×10^7	2.11302×10^6
50	1.61716×10^8	1.40701×10^9	1.79902×10^8	2.99262×10^7	2.29259×10^6
60	1.67733×10^8	1.54719×10^9	2.05467×10^8	2.79554×10^7	2.50321×10^6
70	1.80185×10^8	1.69441×10^9	2.32548×10^8	2.86568×10^7	2.74015×10^6
80	1.96301×10^8	1.85358×10^9	2.6169×10^8	3.05969×10^7	3.00287×10^6
90	2.14991×10^8	2.02764×10^9	2.93273×10^8	3.32089×10^7	3.29261×10^6
100	2.35899×10^8	2.21878×10^9	3.2762×10^8	3.62775×10^7	3.61147×10^6

Graphs for the solution of system (7) using the assumed initial values given in (29) and (30) are also plotted for all the five customers' population to visualize the trend of populations within hundred years, using the software Mathematica, which are shown in Figure 13 to Figure 18. Figures 13 to 17 depict the trend in customers' population in PSB, PB, FB, CB and RP respectively within hundred years. Figure 18 shows the trend in all the populations within hundred years on a single graph. These figures also give the same type of trend in all the five population as observed in Table 1.

Conclusion

Numerical discussion showed that private and foreign banks seem to be progressive in attracting customers as compared to public sector and cooperative banks. Customers in public sector banks and cooperative banksin first half span of hundred years losses their

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interest in dealing with these banks but keep on holding their interest in these banks slowly in the second half span of hundred years. The slight increase in their interest in these banks in the second half span may be due to financial security and guaranteed profits although at less rate provided by the government. The number of people having no banking dealing shows a continuous increasing trend within 100 years span. This may be due to increasing population and resulting unemployment and poverty. From the study presented here, it can also be concluded that public sector banks and co-operative banks need to be more active and techno-savvy to attract young generation. Both of these sectors need to be more efficient, flexible and adaptable to the ever-changing environment. Also, with the aid of real data from banking sectors, the derived model in this study can benefit the banking industry. Furthermore, future study can be carried out to see the other possible non-zero solutions of the homogeneous system of linear equations obtained to determine the equilibrium points in number of customers of various banking sectors.

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