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On Near Semigroups

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Abstract: In this paper we study the properties of near semigroup. We introduce subset of near semigroups in approximation spaces and properties of these near structure are explored. Further, it is proved that near semigroup is a generalization of a semigroup.

Keyword: Near set, Nearness Approximation space, Near Subset of Near semigroup, Nearsemigroup and Sub semigroup.

Mathematical Subject Classification :03E75, 03E99, 20A05, 20E99

1 Introduction

Rough sets were introduced by Z. Pawlak [14] in 1982.An algebraic approach to
rough sets has been given by Iwinski [8]. Later, rough subgroups was introduced by Biswas and Nanda [1]. Rough ideal in a semigroup were introduced by Kuroki [9]. In 2004 and 2006, Davvaz investigated the concept of roughness of rings and modules [2, 3] (and other algebraic approaches to rough sets in ([11],[18],[19],[20]). In 2007, near set theory and nearness approximation spaces was introduced by J. F. Peters as a generalization of rough set theory ([16],[17]). In this theory, Peters utilized the features of objects to definde the nearness of objects [17]and, consequently,the classification of the universal set with respect to the available information of the objects. Nonempty sets are near, provided the sets resemble each other descriptively. It is the resemblance of sets that places near set theory in the fuzzy sciences theory milieu, since membership of a set in a family of near sets depends on a comparison of object descriptions that are usually not exact and such inexact descriptions establish the resemblance of each set in a family of sets that are descriptively near each other. In 2013, Ozturk and Inan [12] combined the soft sets approach with near set theory, which gives rise to the new concepts of soft nearness approximation spaces (SNAS), soft lower and upper approximations.

In 2012, Inan and Ozturk ([5],[6]) investigated the basic concepts of the algebraic structures of the near set theory. They introduced the concept of near groups, weak cosets, near normal subgroups and homomorphism of near groups on nearness approximation spaces. Moreover,

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in 2014, Ozturk et al. [13] introduced near group of weak cosets on nearness approximation spaces. In this article, our aim is to improve the concept of nearness semigroup theory, which extends the notion of a semigroup to include the algebraic structures of near sets. Also, we introduce some propertiesof aproximations and these algebraic structures.

2 Preliminaries

In this section we give some definitions and properties regarding near set [15].

An object description is defined by means of a tuple of function value $\psi(x)$ associated with an object $x \epsilon X$. Assume that $\mathcal{B} \subseteq \mathcal{F}$ is a given set of functions representing features of sample objects $X \subseteq \mathcal{O}$.

Let $\psi_i \epsilon \mathcal{B}$, where $\psi_i: \mathcal{O} \to \mathcal{R}$. In combination the function representing object features provide a basis for an object description $\psi: \mathcal{O} \to \mathcal{R}^{\mathcal{L}}$ a vector containing measurements associated with each functional value $\psi_i(x)$, where the description length $|\psi| = \mathcal{L}$.

Object Description:$\boldsymbol{\psi(x) = (\psi_1(x), \psi_2(x), \psi_3(x), \ldots\ldots \psi_i(x), \ldots\ldots \psi_{\mathcal{L}}(x))}$

Sample objects $X \subseteq \mathcal{O}$ are near each other if and only if the objects have similar descriptions. Recall that each ψ defines a description of an object. Then let Δ_{ψ_i} denote $\Delta_{\psi_i} = |\psi_i(x') - \psi_i(x)|$where $, x' \epsilon \mathcal{O}$. The difference ψ leads to a description of the indiscernibility relation $\sim_{\mathcal{B}_r}$ introduced by peters [15].

Definition 2.1 [16]Let $X, X' \subseteq \mathcal{O}, \mathcal{B} \subseteq \mathcal{F}$. Set X is a near X' if and only if there exists $x \epsilon X, x \epsilon X', \psi_i \epsilon \mathcal{B}$ such that $x \sim_{\psi_i} x'$.

Definition 2.2 [16]Let $x, x' \epsilon \mathcal{O}, \mathcal{B} \subseteq \mathcal{F}$. Then

$$\sim_{\mathcal{B}} = \{(x, x') \epsilon \mathcal{O} \times \mathcal{O} | \forall \psi_i \epsilon \mathcal{B}, \Delta_{\psi_i} = 0\}$$

Is called the indiscernibility relation $\mathcal{O}$, where the description length $i \leq |\psi|$.

Definition 2.3 [7] Let $(\mathcal{O}, \mathcal{F}, \sim_{\mathcal{B}_r}, \mathcal{N}_r, \mathcal{V}_{\mathcal{N}_r})$ be a nearness approximation space and let "." be a binary operation defined $\mathcal{O}$. Let $X \subseteq \mathcal{O}$ and $\mathcal{B}_r \subseteq \mathcal{F}, r \leq |\mathcal{B}|$. A indiscernibility relation $\sim_{\mathcal{B}_r}$on $\mathcal{O}$ is called a complete indiscernibility relation $\sim_{\mathcal{B}_r}$ on perceptual objects$\mathcal{O}$, if $[x]_{\mathcal{B}_r}[y]_{\mathcal{B}_r} = [xy]_{\mathcal{B}_r}$ for all $x, y \epsilon X$.

A nearness approximation space (NAS) is a tuple NAS= $(\mathcal{O}, \mathcal{F}, \sim_{\mathcal{B}_r}, \mathcal{N}_r, \mathcal{V}_{\mathcal{N}_r})$ where the approximation space NAS is defined with a set of perceived objects $\mathcal{O}$, set of probe functions $\mathcal{F}$ representing object features, indiscernibility relation $\sim_{\mathcal{B}_r}$, defined relative to $\mathcal{B}_r \subseteq \mathcal{B} \subseteq \mathcal{F}$, collection of partitions (families of neighbourhoods$\mathcal{N}_r(\mathcal{B})$, and overalp function $\mathcal{V}_{\mathcal{N}_r}$.

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Definition 2.4 [7] A semigroup is an algebraic structure on a nonempty set S together with an associative binary operation. That means, a semigroup is a set S together with a binary operation "." That satisfies:

(1) For all $a, b \in S, a \cdot b \in S$.

(2) For all $a, b, c \in S$, the equation $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ holds in S.

Definition 2.5 [7] A nonempty subset A of a semigroup S is said to be a sub semigroupof S, if $a, b \in A\ for\ all\ a, b \in A\ (i., e)A^2 \subseteq A$

Theorem 2.6[7] Let $(\mathcal{O}, \mathcal{F}, \sim_{\mathcal{B}_r}, \mathcal{N}_r, \mathcal{V}_{\mathcal{N}_r})$ be a nearness approximation space and $X, Y \subset \mathcal{O}$, then the following statement hold;

(1) $\mathcal{N}_r(\mathcal{B})_*(X) \subseteq X \subseteq \mathcal{N}_r(\mathcal{B})^*(X)$

(2) $\mathcal{N}_r(\mathcal{B})^*(X \cup Y) = \mathcal{N}_r(\mathcal{B})^*(X) \cup \mathcal{N}_r(\mathcal{B})^*(Y)$

(3) $\mathcal{N}_r(\mathcal{B})_*(X \cap Y) = \mathcal{N}_r(\mathcal{B})_*(X) \cap \mathcal{N}_r(\mathcal{B})_*(Y)$

(4) $X \subseteq Y\ implies\ \mathcal{N}_r(\mathcal{B})_*(X) \subseteq \mathcal{N}_r(\mathcal{B})_*(Y)$

(5) $X \subseteq Y\ implies \mathcal{N}_r(\mathcal{B})^*(X) \subseteq \mathcal{N}_r(\mathcal{B})^*(Y)$

(6) $\mathcal{N}_r(\mathcal{B})_*(X \cup Y) \supseteq \mathcal{N}_r(\mathcal{B})_*(X) \cup \mathcal{N}_r(\mathcal{B})_*(Y)$

(7) $\mathcal{N}_r(\mathcal{B})^*(X \cap Y) \subseteq \mathcal{N}_r(\mathcal{B})^*(X) \cap \mathcal{N}_r(\mathcal{B})^*(Y)$

Theorem 2.7 [7] Let $(\mathcal{O}, \mathcal{F}, \sim_{\mathcal{B}_r}, \mathcal{N}_r, \mathcal{V}_{\mathcal{N}_r})$ be a nearness approximation space and X and Y are nonempty subsets of perceptual objects $\mathcal{O}$, then

$$\mathcal{N}_r(\mathcal{B})^*(X)\mathcal{N}_r(\mathcal{B})^*(Y) \subseteq \mathcal{N}_r(\mathcal{B})^*(XY)$$

Theorem 2.8 [7] Let $\sim_{\mathcal{B}_r}$be a complete indiscernibility relation on $\mathcal{O}$. If X and Y are nonempty subsets of $\mathcal{O}$, then

$$\mathcal{N}_r(\mathcal{B})_*(X)\mathcal{N}_r(\mathcal{B})_*(Y) \subseteq \mathcal{N}_r(\mathcal{B})_*(XY)$$

3 Near Subset of a Nearsemigroup

Let S be a semigroup. Let $\sim_{\mathcal{B}_r}$ be a congruence on$S$, that is $\sim_{\mathcal{B}_r}$ is an equivalence relation on S such that

$a \sim_{\mathcal{B}_r} b$implies$ax \sim_{\mathcal{B}_r} bx$ and $xa \sim_{\mathcal{B}_r} xb$ for all $x \in S$.

We denote $[a]_{\mathcal{B}_r}$ the $\mathcal{N}_r(\mathcal{B})$ on S is called complete if $[a]_{\mathcal{B}_r}[b]_{\mathcal{B}_r} = [ab]_{\mathcal{B}_r} for\ all\ a, b \in S$

Let A be a nonempty subset of S. Then the sets

$$\mathcal{N}_r(\mathcal{B})_*(A) = \{x \in \mathcal{N}_r(\mathcal{B})_*(S): [x]_{\mathcal{B}_r} \subseteq A\}$$

And

$$\mathcal{N}_r(\mathcal{B})^*(A) = \{x \in \mathcal{N}_r(\mathcal{B})^*(S): [x]_{\mathcal{B}_r} \cap A \neq \theta\}$$

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Are called the lower and upper approximation of A. We denote by $P(S)$ the set of all subsets of S. For a nonempty subset $A$of S.

$$Bnd\mathcal{N}_r(\mathcal{B})(A) = \mathcal{N}_r(\mathcal{B})_*(A), \mathcal{N}_r(\mathcal{B})^*(A)$$

Is called a near set with respect to $\mathcal{N}_r(\mathcal{B})$ or simply a $\mathcal{N}_r(\mathcal{B})$ near subset of $P(S) \times P(S)\ if$

$$(\mathcal{N}_r(\mathcal{B})_*(A) \neq \mathcal{N}_r(\mathcal{B})^*(A))$$

Theorem 3.1 Let $(\mathcal{O}, \mathcal{F}, \sim_{\mathcal{B}_r}, \mathcal{N}_r, \mathcal{V}_{\mathcal{N}_r})$ be a nearness approximation space. If X and Y are nonempty subset of perceptual objects $\mathcal{O}$, then

$$\mathcal{N}_r(\mathcal{B})_*(X) \cup \mathcal{N}_r(\mathcal{B})_*(Y) \subseteq \mathcal{N}_r(\mathcal{B})_*(X \cup Y)$$

Proof: Let $z = \mathcal{N}_r(\mathcal{B})_*(X) \cup \mathcal{N}_r(\mathcal{B})_*(Y)$, then we have $z = x \cup y$ with $x \in \mathcal{N}_r(\mathcal{B})_*(X)$and $y \in \mathcal{N}_r(\mathcal{B})_*(Y)$. Thus there exist element $m, n \in \mathcal{O}$ such that $m \in [x]_{\mathcal{B}_r} \subseteq X\ and\ n \in [y]_{\mathcal{B}_r} \subseteq Y$, consequently $m \in [x]_{\mathcal{B}_r}, n \in [y]_{\mathcal{B}_r}, x \in X$ and $y \in Y$. Since $\sim_{\mathcal{B}_r}$ is an indiscernibility relation in $\mathcal{O}$,$m \cup n \in [x]_{\mathcal{B}_r} \cup [y]_{\mathcal{B}_r} \subseteq [x \cup y]_{\mathcal{B}_r}$. Since $m \cup n \in X \cup Y$ we observe that $m \cup n \in [x \cup y]_{\mathcal{B}_r} \subseteq X \cup Y$ and so $x \cup y \in \mathcal{N}_r(\mathcal{B})_*(X \cup Y)$.Hence $\mathcal{N}_r(\mathcal{B})_*(X) \cup \mathcal{N}_r(\mathcal{B})_*(Y) \subseteq \mathcal{N}_r(\mathcal{B})_*(X \cup Y)$.

Theorem 3.2 Let $(\mathcal{O}, \mathcal{F}, \sim_{\mathcal{B}_r}, \mathcal{N}_r, \mathcal{V}_{\mathcal{N}_r})$ be a nearness approximation space. If X and Y are nonempty subset of perceptual objects $\mathcal{O}$, then

$$\mathcal{N}_r(\mathcal{B})^*(X) \cap \mathcal{N}_r(\mathcal{B})^*(Y) \subseteq \mathcal{N}_r(\mathcal{B})^*(X \cap Y)$$

Proof: Let $z = \mathcal{N}_r(\mathcal{B})^*(X) \cap \mathcal{N}_r(\mathcal{B})^*(Y)$, then we have $z = x \cap y$ with $x \in \mathcal{N}_r(\mathcal{B})^*(X)$and $y \in \mathcal{N}_r(\mathcal{B})^*(Y)$. Thus there exist element $m, n \in \mathcal{O}$ such that $m \in [x]_{\mathcal{B}_r} \cap \neq \emptyset, n \in [y]_{\mathcal{B}_r} \cap \neq \emptyset$, consequently $m \in [x]_{\mathcal{B}_r}, n \in [y]_{\mathcal{B}_r}, x \in X$ and $y \in Y$. Since $\sim_{\mathcal{B}_r}$ is an indiscernibility relation in $\mathcal{O}$,$m \cup n \in [x]_{\mathcal{B}_r} \cap [y]_{\mathcal{B}_r} \subseteq [x \cap y]_{\mathcal{B}_r}$. Since $m \cap n \in X \cap Y$ we observe that $m \cap n \in [x \cap y]_{\mathcal{B}_r} \subseteq X \cap Y$ and so $x \cap y \in \mathcal{N}_r(\mathcal{B})_*(X \cap Y)$.Hence $\mathcal{N}_r(\mathcal{B})_*(X) \cap \mathcal{N}_r(\mathcal{B})_*(Y) \subseteq \mathcal{N}_r(\mathcal{B})_*(X \cap Y)$.

Theorem 3.3 Let $(\mathcal{O}, \mathcal{F}, \sim_{\mathcal{B}_r}, \mathcal{N}_r, \mathcal{V}_{\mathcal{N}_r})$ be a nearness approximation space. If X and Y are nonempty subset of perceptual objects $\mathcal{O}$, then

$$\mathcal{N}_r(\mathcal{B})_*(X \cap Y)(A) = \mathcal{N}_r(\mathcal{B})_*(X)(A) \cap \mathcal{N}_r(\mathcal{B})_*(Y)(A)$$

Proof: Let $z = \mathcal{N}_r(\mathcal{B})_*(X \cap Y)(A)$, then we have $z = x \cap y(A)$ with $x \in \mathcal{N}_r(\mathcal{B})_*(X)\ (A)$and $y \in \mathcal{N}_r(\mathcal{B})_*(Y)(A)$. Thus there exist element $m, n \in \mathcal{O}$ such that $m \in [x]_{\mathcal{B}_r} \subseteq A\ and\ n \in [y]_{\mathcal{B}_r} \subseteq A$, consequently $m \in [x]_{\mathcal{B}_r}, n \in [y]_{\mathcal{B}_r}, x \in X$ and $y \in Y$. Since $\sim_{\mathcal{B}_r}$ is an indiscernibility relation in $\mathcal{O}$,$m \cap n \in [x]_{\mathcal{B}_r}(A) \cap [y]_{\mathcal{B}_r}(A) = [x \cap y]_{\mathcal{B}_r}(A)$.

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Since $m \cap n \in [x \cap y]_{\mathcal{B}_r}(A)$ we observe that $m \cap n \in [x \cap y]_{\mathcal{B}_r}(x \cap y)(A)$ and so $x \cap y \in \mathcal{N}_r(\mathcal{B})_*(X \cap Y)(A)$.Hence $\mathcal{N}_r(\mathcal{B})_*(X)(A) \cap \mathcal{N}_r(\mathcal{B})_*(Y)(A) = (\mathcal{B})_*(X \cap Y)(A)$.

Theorem 3.4 Let $(\mathcal{O}, \mathcal{F}, \sim_{\mathcal{B}_r}, \mathcal{N}_r, \mathcal{V}_{\mathcal{N}_r})$ be a nearness approximation space. If X and Y are nonempty subset of perceptual objects $\mathcal{O}$, then

$$\mathcal{N}_r(\mathcal{B})^*(X \cap Y)(A) \subseteq \mathcal{N}_r(\mathcal{B})^*(X)(A) \cap \mathcal{N}_r(\mathcal{B})^*(Y)(A)$$

Proof: Let $z = \mathcal{N}_r(\mathcal{B})^*(X \cap Y)(A)$, then we have $z = x \cap y(A)$ with $x \in \mathcal{N}_r(\mathcal{B})^*(X)\ (A)$and $y \in \mathcal{N}_r(\mathcal{B})^*(Y)(A)$. Thus there exist element $m, n \in \mathcal{O}$ such that $m \in [x]_{\mathcal{B}_r} \cap A \neq \emptyset\ and\ n \in [y]_{\mathcal{B}_r} \cap A \neq \emptyset$, consequently $m \in [x]_{\mathcal{B}_r}, n \in [y]_{\mathcal{B}_r}, x \in X$ and $y \in Y$. Since $\sim_{\mathcal{B}_r}$ is an indiscernibility relation on $\mathcal{O}$,$m \cap n \in [x]_{\mathcal{B}_r}(A) \cap [y]_{\mathcal{B}_r}(A) \subseteq [x \cap y]_{\mathcal{B}_r}(A)$. Since $m \cap n \in [x \cap y]_{\mathcal{B}_r}(A)$ we observe that $m \cap n \in [x \cap y]_{\mathcal{B}_r}(x \cap y)(A)$ and so $x \cap y \in \mathcal{N}_r(\mathcal{B})^*(X \cap Y)(A)$.Hence $\mathcal{N}_r(\mathcal{B})^*(X)(A) \cap \mathcal{N}_r(\mathcal{B})^*(Y)(A) \subseteq \mathcal{N}_r(\mathcal{B})^*(X \cap Y)(A)$.

4 Near Semigroup

Definition 4.1 [7] Let $(\mathcal{O}, \mathcal{F}, \sim_{\mathcal{B}_r}, \mathcal{N}_r, \mathcal{V}_{\mathcal{N}_r})$ be a nearness approximation space and "." Be a binary operation defined on $\mathcal{O}$.

A subset S of the set of perceptual objects $\mathcal{O}$ is called near semigroup on nearness approximation space, provided the following properties are satisfied.

(1) For all $a, b \in S, a \cdot b \in \mathcal{N}_r(\mathcal{B})^*(S)$

(2) For all $a, b, c \in S$, the equation $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ holds in $\mathcal{N}_r(\mathcal{B})^*(S)$.

Example 4.2 Let$\mathcal{O} = \{o, a, b, c, d, e, f, g, h, i, j, k\}$ be a set of perceptual objects where

$$o = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, b = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, c = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, d = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, e = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},$$

$$f = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, g = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, h = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, i = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, j = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, k = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

For $U = \{[a_{ij}]_{2\times 2} | a_{ij} \in \mathbb{Z}_2\}, r = 1, \mathcal{B} = \{\psi_1, \psi_2, \psi_3\} \subseteq \mathcal{F}$ be a set of probe functions

Probe functions are defined by

$$\psi_1 : \mathcal{O} \to V_1 = \psi_1(A) = |A|$$

$$\psi_2 : \mathcal{O} \to V_2 = \psi_2(A) = Trace(A)$$

$$\psi_3 : \mathcal{O} \to V_3 = \psi_3(A) = |A^2|$$

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Are given in Table 1

	o	a	b	c	d	e	f	g	h	i	j	k
ψ_1	0	1	0	0	0	0	0	0	0	-1	0	0
ψ_2	0	2	1	0	1	1	1	0	1	0	2	1
ψ_3	0	1	0	0	0	0	0	0	0	1	0	0

Let · be a binary operation of perceptual objects on $\mathcal{O}$ as in table 2

	o	a	b	c	d	e	f	g	h	i	j	k
o	o	o	o	o	o	o	o	o	o	o	o	o
a	o	a	b	c	d	e	f	g	h	i	j	k
b	o	b	b	o	o	b	o	g	g	g	k	k
c	o	c	c	c	o	c	o	d	d	d	f	f
d	o	d	o	c	d	c	f	o	d	c	f	o
e	o	e	c	o	o	e	o	h	h	h	i	j
f	o	f	c	c	d	c	f	d	d	f	f	f
g	o	g	o	b	g	b	k	o	g	b	k	o
h	o	h	o	e	h	e	j	o	h	e	j	o
i	o	i	c	b	g	e	k	d	h	a	j	f
j	o	j	e	e	h	e	j	h	h	j	j	j
k	o	k	b	b	g	b	k	g	g	k	k	k

Let $S = \{e, f, h\}$ be a subset of perceptual objects and "$\cdot$" be an operation on $S \subseteq \mathcal{O}$ as in table 3

	e	f	h
e	e	o	h
f	c	f	d
h	e	j	h

$[o]_{\{\psi_1\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_1(o) = 0\}$,

$= \{o, b, c, d, e, f, g, h, j, k\}$

$[a]_{\{\psi_1\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_1(a) = 1\}$,

$= \{a\}$

$[i]_{\{\psi_1\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_1(i) = -1\}$,

$= \{i\}$

Hence we have $\xi_{\{\psi_1\}} = \{[o]_{\{\psi_1\}}, [a]_{\{\psi_1\}}, [i]_{\{\psi_1\}}\}$

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$[o]_{\{\psi_2\}} = \{x' \in \mathcal{O} | \psi_2(x') = \psi_2(o) = 0\},$

$= \{o, c, g, i\}$

$[a]_{\{\psi_2\}} = \{x' \in \mathcal{O} | \psi_2(x') = \psi_2(a) = 2\},$

$= \{a, j\}$

$[b]_{\{\psi_2\}} = \{x' \in \mathcal{O} | \psi_2(x') = \psi_2(b) = 1\},$

$= \{b, d, e, f, h, k\}$

Thus we have $\xi_{\{\psi_2\}} = \{[o]_{\{\psi_2\}}, [a]_{\{\psi_2\}}, [b]_{\{\psi_2\}}\}$

$[o]_{\{\psi_3\}} = \{x' \in \mathcal{O} | \psi_3(x') = \psi_3(o) = 0\},$

$= \{o, b, c, d, e, f, g, h, j, k\}$

$[a]_{\{\psi_3\}} = \{x' \in \mathcal{O} | \psi_3(x') = \psi_3(a) = 1\},$

$= \{a, i\}$

So we obtain that, $\xi_{\{\psi_3\}} = \{[o]_{\{\psi_3\}}, [a]_{\{\psi_3\}}\}$

When, $r = 1$, a set of partitions of $\mathcal{O}$

$\mathcal{N}_1(\mathcal{B}) = \{\xi_{\{\psi_1\}}, \xi_{\{\psi_2\}}, \xi_{\{\psi_3\}}\}.$

Then,

$$\mathcal{N}_r(\mathcal{B})^*(S) = \bigcup_{x:[x]_{\{\psi_i\}} \cap S \neq \emptyset} [x]_{\{\psi_i\}}$$

$$= \{o, b, c, d, e, f, g, h, j, k\}$$

Therefore subset S of perceptual objects $\mathcal{O}$ is a near semigroup.

Example 4.3 Let $\mathcal{O} = \{o, a, b, c, d, e, f, g, h, i, j, k\}$ be a set of perceptual objects where

$$o = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, b = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, c = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, d = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, e = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},$$

$$f = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, g = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, h = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, i = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, j = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, k = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

For $U = \{[a_{ij}]_{2\times 2} | a_{ij} \in \mathbb{Z}_2\}, r = 1, \mathcal{B} = \{\psi_1, \psi_2, \psi_3, \psi_4\} \subseteq \mathcal{F}$ be a set of probe functions are defined by

$$\psi_1 : \mathcal{O} \to V_1 = \psi_1(A) = |A|$$

$$\psi_2 : \mathcal{O} \to V_2 = \psi_2(A) = |A^2|$$

$$\psi_3 : \mathcal{O} \to V_3 = \psi_3(A) = Trace(A)$$

$$\psi_4 : \mathcal{O} \to V_4 = \psi_4(A) = |A^T|$$

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Are given in Table 4

	o	a	b	c	d	e	f	g	h	i	j	k
ψ_1	0	1	0	0	0	0	0	0	0	-1	0	0
ψ_2	0	1	0	0	0	0	0	0	1	0	0	0
ψ_3	0	2	1	0	1	1	1	0	1	0	2	1
ψ_4	0	1	0	-1	0	0	0	-1	0	-1	0	0

Let $\cdot$ be a binary operation of perceptual objects on $\mathcal{O}$ as in table 5

	o	a	b	c	d	e	f	g	h	i	j	k
o	o	o	o	o	o	o	o	o	o	o	o	o
a	o	a	b	c	d	e	f	g	h	i	J	k
b	o	b	b	o	o	b	o	g	g	g	k	k
c	o	c	c	c	o	c	o	d	d	d	f	f
d	o	d	o	c	d	c	f	o	d	c	f	o
e	o	e	c	o	o	e	o	h	h	h	i	j
f	o	f	c	c	d	c	f	d	d	f	f	f
g	o	g	o	b	g	b	k	o	g	b	k	o
h	o	h	o	e	h	e	j	o	h	e	j	o
i	o	i	c	b	g	e	k	d	h	a	j	f
j	o	j	e	e	h	e	j	h	h	j	j	j
k	o	k	b	b	g	b	k	g	g	k	k	k

Let $S = \{e, f, h\}$ be a subset of perceptual objects and "$\cdot$" be an operation on $S \subseteq \mathcal{O}$ as in table 6

	e	f	h
e	e	o	h
f	c	f	d
h	e	j	h

$[o]_{\{\psi_1,\psi_2\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_2(x') = \psi_1(o) = \psi_2(o) = 0\},$

$= \{o, b, c, d, e, f, g, j, k\}$

$[a]_{\{\psi_1,\psi_2\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_2(x') = \psi_1(a) = \psi_2(a) = 1\},$

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$$= \{a\}$$

Hence we have $\xi_{\{\psi_1,\psi_2\}} = \{[o]_{\{\psi_1,\psi_2\}}, [a]_{\{\psi_1,\psi_2\}}\}$

$$[o]_{\{\psi_2,\psi_3\}} = \{x' \in \mathcal{O} | \psi_2(x') = \psi_3(x') = \psi_2(o) = \psi_3(o) = 0\},$$
$$= \{o, c, g, i\}$$
$$[h]_{\{\psi_2,\psi_3\}} = \{x' \in \mathcal{O} | \psi_2(x') = \psi_3(x') = \psi_2(h) = \psi_3(h) = 1\},$$
$$= \{h\}$$

Thus we have $\xi_{\{\psi_2,\psi_3\}} = \{[o]_{\{\psi_2,\psi_3\}}, [h]_{\{\psi_2,\psi_3\}}\}$

$$[o]_{\{\psi_3,\psi_4\}} = \{x' \in \mathcal{O} | \psi_3(x') = \psi_4(x') = \psi_3(o) = \psi_4(o) = 0\},$$
$$= \{o\}$$

Thus we have $\xi_{\{\psi_3,\psi_4\}} = \{[o]_{\{\psi_3,\psi_4\}}\}$

$$[o]_{\{\psi_1,\psi_3\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_3(x') = \psi_1(o) = \psi_3(o) = 0\},$$
$$= \{o, c, g\}$$

Thus we have $\xi_{\{\psi_1,\psi_3\}} = \{[o]_{\{\psi_1,\psi_3\}}\}$

$$[o]_{\{\psi_2,\psi_4\}} = \{x' \in \mathcal{O} | \psi_2(x') = \psi_4(x') = \psi_2(o) = \psi_4(o) = 0\},$$
$$= \{o, b, d, e, f, j, k\}$$
$$[a]_{\{\psi_2,\psi_4\}} = \{x' \in \mathcal{O} | \psi_2(x') = \psi_4(x') = \psi_2(a) = \psi_4(a) = 1\},$$
$$= \{a\}$$

Thus we have $\xi_{\{\psi_1,\psi_3\}} = \{[o]_{\{\psi_2,\psi_4\}}, [a]_{\{\psi_2,\psi_4\}}\}$

$$[o]_{\{\psi_1,\psi_4\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_4(x') = \psi_1(o) = \psi_4(o) = 0\},$$
$$= \{o, b, d, e, h, f, j, k\}$$
$$[a]_{\{\psi_1,\psi_4\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_4(x') = \psi_1(a) = \psi_4(a) = 1\},$$
$$= \{a\}$$
$$[i]_{\{\psi_1,\psi_4\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_4(x') = \psi_1(i) = \psi_4(i) = -1\},$$
$$= \{i\}$$

Hence we obtain that $\xi_{\{\psi_1,\psi_4\}} = \{[o]_{\{\psi_4,\psi_4\}}, [a]_{\{\psi_1,\psi_4\}}, [i]_{\{\psi_1,\psi_4\}}\}$

When, $r = 2$, a set of partitions of $\mathcal{O}$ is

$$\mathcal{N}_2(\mathcal{B}) = \{\xi_{\{\psi_1,\psi_2\}}, \xi_{\{\psi_2,\psi_3\}}, \xi_{\{\psi_3,\psi_4\}}, \xi_{\{\psi_1,\psi_3\}}, \xi_{\{\psi_2,\psi_4\}}, \xi_{\{\psi_1,\psi_4\}}\}.$$

Then,

$$\mathcal{N}_r(\mathcal{B})^*(S) = \bigcup_{x:[x]_{\{\psi_i,\psi_j\}} \cap S \neq \emptyset} [x]_{\{\psi_i,\psi_j\}}$$
$$= \{o, b, c, d, e, f, g, h, j, k\}$$

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Therefore subset S of perceptual objects $\mathcal{O}$ is a near semigroup.

Definition 4.4 Let S be a nearsemigroupon $\mathcal{O}$,$\mathcal{B}_r \subseteq \mathcal{F}$ where $r = |\mathcal{B}|$ and $\mathcal{B} \subseteq \mathcal{F}, \sim_{\mathcal{B}_r}$ be a indiscernibility relation on $\mathcal{O}$. Then $\sim_{\mathcal{B}_r}$ is called a congruence indiscernibility relation on nearness semigroup$S$, if $x \sim_{\mathcal{B}_r} y$, where $x, y \in S$ implies $xa \sim_{\mathcal{B}_r} ya$ and $ax \sim_{\mathcal{B}_r} ay$, for all $a \in S$

Lemma 4.5 Let S be a near semigroup if $\sim_{\mathcal{B}_r}$ is a congruence indiscernibility relation on S, then $[x]_{\mathcal{B}_r}[y]_{\mathcal{B}_r} \subseteq [xy]_{\mathcal{B}_r}$ for all $x, y \in S$.

Proof: Let $z = [x]_{\mathcal{B}_r}[y]_{\mathcal{B}_r}$ in this case, $z = ab; a \in [x]_{\mathcal{B}_r}, b \in [y]_{\mathcal{B}_r}$.from here $x \sim_{\mathcal{B}_r} a$ and $y \sim_{\mathcal{B}_r} b$, and so we have $xy \sim_{\mathcal{B}_r} ay$ and $ay \sim_{\mathcal{B}_r} ab$ by hypothesis. Thus, $xy \sim_{\mathcal{B}_r} ab \Rightarrow z = ab \in [xy]_{\mathcal{B}_r}, so\ [x]_{\mathcal{B}_r}[y]_{\mathcal{B}_r}$, is obtained

Definition 4.6 Let S be a near semigroup,$\mathcal{B}_r \subseteq \mathcal{F}$ where $r = |\mathcal{B}|$ and $\mathcal{B} \subseteq \mathcal{F}, \sim_{\mathcal{B}_r}$ be a indiscernibility relation on $\mathcal{O}$. Then $\sim_{\mathcal{B}_r}$ is complete congruence indiscernibility relation on nearness semigroup S, if $[x]_{\mathcal{B}_r}[y]_{\mathcal{B}_r} = [xy]_{\mathcal{B}_r}$, for all $x, y \in S$.

Let S be a nearness semigroup. Let $XY = \{xy | x \in X\ and\ y \in Y\}$ where subsets X and $Y$of S.

Lemma 4.7 Let S be a nearness semigroup. The following properties hold:

(1) If $X, Y \subseteq S$, then $(\mathcal{N}_r(\mathcal{B})^*(X)\mathcal{N}_r(\mathcal{B})^*(Y) \subseteq \mathcal{N}_r(\mathcal{B})^*(XY)$.

(2) If $XY \subseteq S$, and $\sim_{\mathcal{B}_r}$is a complete congruence indiscernibility relation on S, then $(\mathcal{N}_r(\mathcal{B})_*(X)\mathcal{N}_r(\mathcal{B})_*(Y) \subseteq \mathcal{N}_r(\mathcal{B})_*(XY)$.

Proof:

(1) Let $x \in \mathcal{N}_r(\mathcal{B})^*(X)\mathcal{N}_r(\mathcal{B})^*(Y)$. We have $x = ab; a \in \mathcal{N}_r(\mathcal{B})^*(X)$
$b \in \mathcal{N}_r(\mathcal{B})^*(Y). a \in \mathcal{N}_r(\mathcal{B})^*(X) \Rightarrow [a]_{\mathcal{B}_r} \cap X \neq \emptyset. y \in [a]_{\mathcal{B}_r} \cap X \Rightarrow y \in [a]_{\mathcal{B}_r}andy \in x$.Likewise, $b \in \mathcal{N}_r(\mathcal{B})^*(Y) \Rightarrow [b]_{\mathcal{B}_r} \cap Y \neq \emptyset. z \in [b]_{\mathcal{B}_r} \cap Y \Rightarrow z \in [b]_{\mathcal{B}_r}$and $z \in y$ since $w \in yz \in [a]_{\mathcal{B}_r}[b]_{\mathcal{B}_r} \subseteq [ab]_{\mathcal{B}_r}$, we get $w \in [ab]_{\mathcal{B}_r}$,and $w \in XY$. Thus $w \in [ab]_{\mathcal{B}_r} \cap XY \Rightarrow [ab]_{\mathcal{B}_r} \cap [XY] \neq \emptyset$ and so $ab = x \in \mathcal{N}_r(\mathcal{B})^*(XY)$.

(2) Let $x \in \mathcal{N}_r(\mathcal{B})_*(X)\mathcal{N}_r(\mathcal{B})_*(Y)$. We have $x = ab; a \in \mathcal{N}_r(\mathcal{B})_*(X)$,
$b \in \mathcal{N}_r(\mathcal{B})_*(Y)$ In this case, $a \in \mathcal{N}_r(\mathcal{B})_*(X) \Rightarrow [a]_{\mathcal{B}_r} \subseteq X$and $b \in \mathcal{N}_r(\mathcal{B})_*(Y) \Rightarrow [b]_{\mathcal{B}_r} \subseteq Y$, so we obtain $[a]_{\mathcal{B}_r}[b]_{\mathcal{B}_r} \subseteq XY$ on the other hand since $[ab]_{\mathcal{B}_r} = [a]_{\mathcal{B}_r}[b]_{\mathcal{B}_r} \subseteq XY$. Thus$[ab]_{\mathcal{B}_r} \subseteq XY$, and so $ab = x \in \mathcal{N}_r(\mathcal{B})_*(XY)$.

Theorem 4.8 Let S be a nearness semigroup, $\sim_{\mathcal{B}_r}$a complete congruence indiscernibility relation on S, and XY two nonempty subsets of S. The following properties hold.

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(1) $(\mathcal{N}_r(\mathcal{B})^*(X)\mathcal{N}_r(\mathcal{B})^*(Y)) = \mathcal{N}_r(\mathcal{B})^*(XY)$.

(2) $\left(\mathcal{N}_r(\mathcal{B})_*(X)\mathcal{N}_r(\mathcal{B})_*(Y)\right) = \mathcal{N}_r(\mathcal{B})_*(XY)$

Proof: The proof of (1) and (2) is straight forward by the similar way to the proof of lemma (4.7).

5 Sub Nearsemigroup

Definition 5.1 [11] Let $(\mathcal{O}, \mathcal{F}, \sim_{\mathcal{B}_r}, \mathcal{N}_r, \mathcal{V}_{\mathcal{N}_r})$ be a nearness approximation space $\mathcal{O}$. Let S be a near semigroup and A be a nonempty subset of S is said to be a sub near semigroup S, if $ab \in \mathcal{N}_r(\mathcal{B})^*(A)$ for all $a, b \in A$ (i.,e)$AA \in \mathcal{N}_r(\mathcal{B})^*(A)$

Example 5.2 Let $\mathcal{O} = \{o, a, b, c, d, e, f\}$ be a set of perceptual objects where,

$o = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, b = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, c = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, d = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$,

$e = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, f = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ for $U = \{[a_{ij}]_{2\times 2} | a_{ij} \in \mathbb{Z}_2\}, r = 1, \mathcal{B} = \{\psi_1, \psi_2, \psi_3\} \subseteq \mathcal{F}$ be a set of probe functions

Probe functions are defined by

	o	a	b	c	d	e	f
ψ_1	0	1	0	0	0	0	0
ψ_2	0	2	1	0	1	1	1
ψ_3	0	1	0	-1	0	0	0

$$\psi_1 : \mathcal{O} \to V_1 = \psi_1(A) = |A|$$
$$\psi_2 : \mathcal{O} \to V_2 = \psi_2(A) = Trace(A)$$
$$\psi_3 : \mathcal{O} \to V_3 = \psi_3(A) = |A^T|$$

Are given in Table 7

Let $\cdot$ be a binary operation of perceptual objects on $\mathcal{O}$ as in table 8

	o	a	b	c	d	e	f
o	o	o	o	o	o	o	o
a	o	a	b	c	d	e	f
b	o	b	b	o	o	b	o
c	o	c	c	c	o	c	o
d	o	d	o	c	d	c	f
e	o	e	c	o	o	e	o
f	o	f	c	c	d	c	f

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Let $S = \{b, c, d\} \subset \mathcal{O}$ and $A = \{c\} \subset S$ be a subset of perceptual objects and " $\cdot$ " be a binary operation as in table 9

	b	c	d
b	b	o	o
c	c	c	o
d	o	c	d

$[o]_{\{\psi_1\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_1(o) = 0\}$,

$= \{o, b, c, d, e, f\}$

$[a]_{\{\psi_1\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_1(a) = 1\}$,

$= \{a\}$

Hence we have $\xi_{\{\psi_1\}} = \{[o]_{\{\psi_1\}}, [a]_{\{\psi_1\}}\}$

$[o]_{\{\psi_2\}} = \{x' \in \mathcal{O} | \psi_2(x') = \psi_2(o) = 0\}$,

$= \{o, c\}$

$[a]_{\{\psi_2\}} = \{x' \in \mathcal{O} | \psi_2(x') = \psi_2(a) = 2\}$,

$= \{a\}$

$[b]_{\{\psi_2\}} = \{x' \in \mathcal{O} | \psi_2(x') = \psi_2(b) = 1\}$,

$= \{b, d, e, f\}$

Thus we have $\xi_{\{\psi_2\}} = \{[o]_{\{\psi_2\}}, [a]_{\{\psi_2\}}, [b]_{\{\psi_2\}}\}$

$[o]_{\{\psi_3\}} = \{x' \in \mathcal{O} | \psi_3(x') = \psi_3(o) = 0\}$,

$= \{o, b, d, e, f\}$

$[a]_{\{\psi_3\}} = \{x' \in \mathcal{O} | \psi_3(x') = \psi_3(a) = 1\}$,

$= \{a\}$

$[c]_{\{\psi_3\}} = \{x' \in \mathcal{O} | \psi_3(x') = \psi_3(c) = -1\}$,

$= \{c\}$

So we obtain that, $\xi_{\{\psi_3\}} = \{[o]_{\{\psi_3\}}, [a]_{\{\psi_3\}}, [c]_{\{\psi_3\}}\}$

When, $r = 1$, a set of partition of $\mathcal{O}$ is

$\mathcal{N}_1(\mathcal{B}) = \{\xi_{\{\psi_1\}}, \xi_{\{\psi_2\}}, \xi_{\{\psi_3\}}\}$.

Then,

$$\mathcal{N}_r(\mathcal{B})^*(S) = \bigcup_{x:[x]_{\{\psi_i\}} \cap S \neq \emptyset} [x]_{\{\psi_i\}}$$

$$= \{o, b, c, d, e, f\}$$

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Furthermore,

$$\mathcal{N}_r(\mathcal{B})^*(A) = \bigcup_{x:[x]_{\{\psi_i\}} \cap A \neq \emptyset} [x]_{\{\psi_i\}}$$

$$= \{o, b, c, d, e, f\}$$

Therefore since $AA \subseteq \mathcal{N}_r(\mathcal{B})^*(A), A$ is a sub nearsemigroup of S.

Example 5.3 Let $\mathcal{O} = \{o, a, b, c, d, e, f\}$ be a set of perceptual objects where,

$o = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, b = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, c = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, d = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$

$e = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, f = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ for $U = \{[a_{ij}]_{2\times 2} | a_{ij} \in \mathbb{Z}_2\}, r = 1, \mathcal{B} = \{\psi_1, \psi_2, \psi_3, \psi_4\} \subseteq \mathcal{F}$ be a set of probe functions

Probe functions are defined by

$$\psi_1 : \mathcal{O} \to V_1 = \psi_1(A) = |A^T|$$

$$\psi_2 : \mathcal{O} \to V_2 = \psi_2(A) = |A^2|$$

$$\psi_3 : \mathcal{O} \to V_3 = \psi_3(A) = Trace\ (A)$$

$$\psi_4 : \mathcal{O} \to V_4 = \psi_4(A) = |A|$$

Are given in Table 10

	o	a	b	c	d	e	f
ψ_1	0	1	0	-1	0	0	0
ψ_2	0	1	0	0	0	0	0
ψ_3	0	2	1	0	1	1	1
ψ_4	0	1	0	0	0	0	0

Let $\cdot$ be a binary operation of perceptual objects on $\mathcal{O}$ as in table 11

	o	a	b	c	d	e	f
o	o	o	o	o	o	o	o
a	o	a	b	c	d	e	f
b	o	b	b	o	o	b	o
c	o	c	c	c	o	c	o
d	o	d	o	c	d	c	f
e	o	e	c	o	o	e	o
f	o	f	c	c	d	c	f

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Let $S = \{b, c, d\} \subset \mathcal{O}$ and $A = \{d\} \subset S$ be a subset of perceptual objects and "$\cdot$" be a binary operation as in table 12

	b	c	d
b	b	o	o
c	c	c	o
d	o	c	d

$[o]_{\{\psi_1,\psi_2\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_2(x') = \psi_1(o) = \psi_2(o) = 0\},$

$= \{o, b, d, e, f\}$

$[a]_{\{\psi_1,\psi_2\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_2(x') = \psi_1(a) = \psi_2(a) = 1\},$

$= \{a\}$

Hence we have $\xi_{\{\psi_1,\psi_2\}}= \{[o]_{\{\psi_1,\psi_2\}}, [a]_{\{\psi_1,\psi_2\}}\}$

$[o]_{\{\psi_2,\psi_3\}} = \{x' \in \mathcal{O} | \psi_2(x') = \psi_3(x') = \psi_2(o) = \psi_3(o) = 0\},$

$= \{o, c\}$

Thus we have $\xi_{\{\psi_2,\psi_3\}}= \{[o]_{\{\psi_2,\psi_3\}}\}$

$[o]_{\{\psi_3,\psi_4\}} = \{x' \in \mathcal{O} | \psi_3(x') = \psi_4(x') = \psi_3(o) = \psi_4(o) = 0\},$

$= \{o, c\}$

Thus we have $\xi_{\{\psi_3,\psi_4\}}= \{[o]_{\{\psi_3,\psi_4\}}\}$

$[o]_{\{\psi_1,\psi_3\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_3(x') = \psi_1(o) = \psi_3(o) = 0\},$

$= \{o\}$

Thus we have $\xi_{\{\psi_1,\psi_3\}}= \{[o]_{\{\psi_1,\psi_3\}}\}$

$[o]_{\{\psi_2,\psi_4\}} = \{x' \in \mathcal{O} | \psi_2(x') = \psi_4(x') = \psi_2(o) = \psi_4(o) = 0\},$

$= \{o, b, c, d, e, f\}$

$[a]_{\{\psi_2,\psi_4\}} = \{x' \in \mathcal{O} | \psi_2(x') = \psi_4(x') = \psi_2(a) = \psi_4(a) = 1\},$

$= \{a\}$

Thus we have $\xi_{\{\psi_1,\psi_3\}}= \{[o]_{\{\psi_2,\psi_4\}}, [a]_{\{\psi_2,\psi_4\}}\}$

$[o]_{\{\psi_1,\psi_4\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_4(x') = \psi_1(o) = \quad _4(o) = 0\},$

$= \{o, b, d, e, f\}$

$[a]_{\{\psi_1,\psi_4\}} = \{x' \in \mathcal{O} | \psi_1(x') = \psi_4(x') = \psi_1(a) = \psi_4(a) = 1\},$

$= \{a\}$

Hence we obtain that $\xi_{\{\psi_1,\psi_4\}}= \{[o]_{\{\psi_4,\psi_4\}}, [a]_{\{\psi_1,\psi_4\}}\}$

When, $r = 2$, a set of partitions of $\mathcal{O}$ is

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$\mathcal{N}_2(\mathcal{B}) = \{\xi_{\{\psi_1,\psi_2\}}, \xi_{\{\psi_2,\psi_3\}}, \xi_{\{\psi_3,\psi_4\}}, \xi_{\{\psi_1,\psi_3\}}, \xi_{\{\psi_2,\psi_4\}}, \xi_{\{\psi_1,\psi_4\}}\}$.

Then,

$$\mathcal{N}_r(\mathcal{B})^*(S) = \bigcup_{x:[x]_{\{\psi_i,\psi_j\}} \cap S \neq \emptyset} [x]_{\{\psi_i,\psi_j\}}$$

$$= \{o, b, c, d, e, f\}$$

Furthermore,

$$\mathcal{N}_r(\mathcal{B})^*(A) = \bigcup_{x:[x]_{\{\psi_i,\psi_j\}} \cap A \neq \emptyset} [x]_{\{\psi_i,\psi_j\}}$$

$$= \{o, b, c, d, e, f\}$$

Therefore since $AA \subseteq \mathcal{N}_r(\mathcal{B})^*(A), A$ is a sub nearsemigroupof S.

Theorem 5.4 Let $\left(\mathcal{O}, \mathcal{F}, \sim_{\mathcal{B}_r}, \mathcal{N}_r, \mathcal{V}_{\mathcal{N}_r}\right)$ be an approximation and $\cdot$ be a binary operation defined on $\mathcal{O}$. Let A_1 and A_2 be two sub near semigroups of the near semigroup S. A sufficient condition for intersection of two sub near semigroups of a near semigroup to be a sub near semigroup is $\mathcal{N}_r(\mathcal{B})^* A_1 \cup \mathcal{N}_r(\mathcal{B})^* A_2 = \mathcal{N}_r(\mathcal{B})^*(A_1 \cup A_2)$.

Proof: Suppose $A_1$and A_2 are two sub near semigroup of S. It is obvious that $A_1 \cup A_2 \subset S$. Consider $x, y \in A_1 \cup A_2$. Because $A_1$and A_2 aresub near semigroups, we have $xy \in \mathcal{N}_r(\mathcal{B})^* A_1, xy \in \mathcal{N}_r(\mathcal{B})^* A_2$ (i.,e) $xy \in \mathcal{N}_r(\mathcal{B})^* A_1 \cup \mathcal{N}_r(\mathcal{B})^* A_2$. Assuming $\mathcal{N}_r(\mathcal{B})^* A_1 \cup \mathcal{N}_r(\mathcal{B})^* A_2 = \mathcal{N}_r(\mathcal{B})^*(A_1 \cup A_2)$,we have $xy \in \mathcal{N}_r(\mathcal{B})^*(A_1 \cup A_2)$. Thus $A_1 \cup A_2$ is a sub near semigroup of S.

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