How to Cite:
Savita Garg and Neetu Rani (Dec 2016). A Comparative Study of Nine Mathematical Models to Analyse Economic Phenomena
International Journal of Economic Perspectives, 10(1), 92-102
Retrieved from: http://ijeponline.com/index.php/journal

A Comparative Study of Nine Mathematical Models to Analyse Economic Phenomena

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Abstract

The study investigates the economic aspects of nine foundational mathematical models: Linear Regression, Taylor Series, Exponential Growth, Compounded Interest Formula, Logistic Growth, Solow-Swan Growth Model, Harrod-Domar Model, Cobb-Douglas Production Function, and the Phillips Curve. The applications of these models in economic forecasting, elucidating their commonalities and unique characteristics have been explored. Key assumptions sustaining these models are dissected, and the limitations inherent in each are critically evaluated. The article compares their roles in predicting diverse economic phenomena, from inflation rates to economic growth, and from investment requirements to production levels. To deal with general economic phenomena, advanced mathematical expressions for five models have also been provided. The study emphasizes the significance of understanding these models' strengths and weaknesses in making economic forecasts and concludes that no single model provides a panacea for predicting all economic phenomena.

Key Words: Mathematical models; Economic growth; Linear regression; Exponential growth; Logistic growth; Production function.

Introduction

Mathematical models play an integral role in the field of economics, enabling economists to conceptualize, analyze, and predict complex economic phenomena (Morgan and Knuuttila, 2012). By transforming abstract theories into quantifiable equations and systems, these models illuminate relationships between economic variables, facilitating rigorous analysis. Over the centuries, numerous mathematical models have been developed to understand economic systems. The elementary forms of mathematical models were observed in ancient civilizations, with the Egyptians and Babylonians using arithmetic and geometry for tax calculation and land measurement (Joseph, 1987). However, the systematic use of mathematical models in economic analysis commenced in the 19th century (David, 1975).
In 1838, the Cournot model of oligopoly introduced the application of calculus in economic theory (Rosser, 2002). This was followed by the development of Leon Walras's general equilibrium model in the 1870s, which used advanced mathematics

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Submitted: 26 Oct 2016, Revised: 09 Nov 2016, Published: 30 Dec 2016

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to explain the interaction of supply and demand in various markets (Friedman, 1955).In the early 20th century, the concept of exponential growth began to gain popularity in economics, modeling constant rate phenomena like population growth, inflation, and investments. Around the same time, the Cobb-Douglas production function was developed by Charles Cobb and Paul Douglas in 1928 (Douglas, 1976). This model remains fundamental in understanding the relationship between labor and capital in the production process. The mid-20th century saw the introduction of several key mathematical models in economics. The Harrod-Domar growth model was introduced in the 1940s, providing a new approach to understanding economic growth and development (Button, 2000). The Solow-Swan growth model followed a decade later in 1956, contributing to an enhanced understanding of economic growth factors (Dimand and Spencer, 2009). In 1958, A.W. Phillips introduced the Phillips Curve, establishing a relationship between inflation and unemployment (Humphrey, 1985). This model continues to guide macroeconomic policy and discussions on the trade-off between inflation and unemployment. The 1970s brought a more pronounced emphasis on econometrics, with the application of linear regression and other statistical models (Anselin, 1988). These models are used by economists to estimate and test hypotheses about the relationships between economic variables.
More recently, advancements in computation have led to the development of complex mathematical models, such as game theory models, agent-based models, and network models, which address a range of economic phenomena from strategic interactions to complex market dynamics (Nolan, 2009).

Scope of the Paper

The study will investigate a variety of mathematical models and concepts prevalent in economics, analyzing their underlying assumptions, applications, commonalities, and limitations. The scope covers nine mathematical models. Examining these models will assess their individual and comparative roles in predicting economic phenomena and understanding the complexity of economic systems. The aim is to study how these mathematical tools are instrumental in understanding, interpreting, and predicting economic behaviors.

Research Methodology

This study explores nine mathematical models for analyzing economic phenomena. Utilizing theoretical exploration, the study involves a review of model theories, assumptions, and their application to selected economic indicators. Data, sourced from reputable databases, undergoes rigorous analysis for each model, interpreting results within the models' theoretical contexts. Findings aim to highlight each model's applicability and limitations in economic analysis, thereby informing future research. All sources and data are properly credited, maintaining ethical standards.

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Mathematical Concepts/Models Used for Analysis of Economic Growth

Economic growth can be analysed using mathematical concepts. An economist witnesses mathematical expressions and concepts at every next step while studying economic growth. A few mathematical concepts have been listed in Table 1.

Table 1: Mathematical Concepts/Models Used in Economic Growth Study

Sr. No.	Concept/Model	Mathematical Expression	Symbols Used
1	Linear Regression (EM-1)	$D_V=A+B\ I_V$	D_V: dependent variable; I_V: independent variable; A: D_V-intercept; B: slope.
2	Taylor Series (EM-2)	$G(t)=G(t_o)+(t-t_o)\ G'(t_o)+((t-t_o)^2/2!)\ G''(t_o)+\ldots$	G(t): infinitely differentiable function at t; G': first derivative of G; G'': second derivative of G.
3	Exponential Growth (EM-3)	$Q=Q_o\exp(R_gt)$	Q: quantity of an object; Q_o: initial quantity of the object; R_g: growth rate; t: time.
4	Compounded Interest Formula (EM-4)	$M_A=P_A\ (1+R_A/N)^{N\ t}$	MA: money accumulated; PA: principal amount; RA: annual interest rate; N: number of times the interest is compounded per year; t: number of years.
5	Logistic Growth (EM-5)	$dP/dt=I\ P\ (1-P/C_c)$	P: population size; I: intrinsic rate of increase; C_c: carrying capacity; dP/dt: rate of population growth.
6	Solow-Swan Growth Model (EM-6)	$O/W=(C/W)^{Co}$	W: number of workers; O: output; C: capital; Co: capital share of output.
7	Harrod-Domar Model (EM-7)	$I_R=(S_r/(C_A-S_r))\ G$	I_R: investment required; S_r: savings ratio; C_A: capital-output ratio; G: gross domestic product
8	Cobb-Douglas Production Function (EM-8)	$P=F_P\ C\ e_o\ L\ (1-e_o)$	P: production; F_P: total factor productivity; C: capital input; L: labour input; e_o: output elasticity of capital; 1-e_o: output elasticity of labour.
9	Phillips Curve (EM-9)	$I=E[I]-A\ (U_a-U_N)$	I: inflation; E[I]: expected inflation; U_a: actual unemployment; U_N: natural rate of unemployment; A: a positive constant.

The economic perspectives of these mathematical concepts/models represented by

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EM-i (1 ≤ i ≤ 9) are given as-

- EM-1: Linear regression is a basic predictive analytics technique (Yatchew, 1998). It is used for estimating the relationships among variables. Table 1 shows the simplest form of the linear regression equation with two variables.
- EM-2: Taylor series is basically used in economics to approximate functions by a finite series (Viscusi & Evans, 1990).
- EM-3: Exponential growth is a pattern of data that shows greater increases with passing time, creating the curve of an exponential function. McKenzie and Liersch (2011) observed experimentally that people generally underestimate exponential growth, particularly in retirement savings context, perceiving it as linear. They concluded that misjudgment results in under-saving and postponing savings. They proposed that highlighting exponential growth potential in savings can motivate increased retirement contributions, effectively encouraging better retirement preparation.
- EM-4: Compounded interest formula calculates the future value of an investment or loan, which is compounded over time. Thus, this formula can be viewed as a predictive model for calculating accumulated interest over a certain period (Pournara, 2011).
- EM-5: Logistic growth is a common S-shaped growth model that describes slowed growth as carrying capacity is reached. It is widely researched and applied across various socio-technical systems due to its effective representation of growth dynamics in these areas (Meyer, 1994). Malthusian used the logistic growth model to describe how population growth can outpace the growth of food supply until starvation, disease, or war reduces the population to sustainable levels (Voigtländer&Voth, 2013).
- EM-6: The Solow-Swan model is a neoclassical growth model where the level of output per worker is a function of capital per worker (Guerrini, 2006). It describes conditions for a steady state of economic growth and also forecasts long-term economic growth by assuming constant savings and depreciation rates.
- EM-7: The Harrod-Domer model is a classic economic mathematical model that proposes the investment needed for a nation to achieve a target growth rate (Masoud, 2014).
- EM-8: Cobb-Douglas production function is a particular form of the production function. This is widely used to represent the technological relationship between the amounts of two or more inputs (e.g., capital and labor) and the amount of output (i.e., production) (Zellner et al., 1966; Douglas, 1976).
- EM-9: The Phillips curve is an economic concept developed by A.W. Phillips in 1958 showing that inflation and unemployment have a stable and inverse relationship. It represents a policy trade-off for optimal inflation and unemployment levels. Alisa (2015) used this concept to explore the

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relationship between unemployment and inflation, key facets of a market economy that profoundly impact living standards. Also, Alisa explained varying economic perspectives on the Phillips curve, scrutinizing both short-term and long-term curves using Russian statistical data, to understand their interplay better.

Commonalities in the Models

All models predict the growth or change in a specific parameter, though they address different aspects of these changes. The common elements among these models include-

i. Estimation of relationships (e.g., linear regression, and Cobb-Douglas production function)
ii. Prediction and forecasting (e.g., compounded interest formula, and Solow-Swan model)
iii. Understanding underlying mechanisms (e.g., Harrod-Domar model, and Phillips curve)
iv. Assumptions (e.g., logistic growth model, and Solow-Swan model)
v. Quantitative analysis
vi. Optimization and equilibrium (e.g., Solow-Swan model, and Phillips curve)
vii. Incorporation of time (e.g., exponential growth model, and compounded interest formula)

Mathematical Assumptions and Limitations of the Models

The mathematical models are useful from the economics perspective, yet all have limitations that must be considered when interpreting their results. The noteworthy limitations of each model are given in Table 2.

Table 2: Assumptions and Limitations of the Models

Sr. No.	Model	Assumptions and Limitations
1	EM-1	(i) The model assumes a linear relationship between dependent and independent variables, which might not always hold in real-world scenarios. (ii) It is sensitive to outliers and assumes homoscedasticity (constant variance of the errors), which is not always the case.
2	EM-2	(i) The mathematical technique approximates around a certain point, so its accuracy is compromised for values farther from the point. (ii) The function must be differentiable to construct its Taylor series.
3	EM-3	(i) The model assumes a constant growth rate over time and ignores potential limiting factors. In the real world, rarely any phenomenon can grow exponentially indefinitely.
4	EM-4	(i) The mathematical technique assumes a fixed interest rate and

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		regular compounding intervals. In the real scenario, interest rates can change, and compound interest calculations can become complex with varying intervals.
5	EM-5	(i) The model assumes a constant carrying capacity, i.e., maximum achievable growth, which might not hold true in dynamic environments. (ii) The model assumes a symmetric growth around the inflection point, which may not be accurate.
6	EM-6	(i) The model assumes constant returns to scale and a constant saving rate, which may not reflect real-world scenarios. (ii) The model does not consider other sources of economic growth like institutional factors or human capital.
7	EM-7	(i) The model assumes that the economy is always at full employment and that saving ratios and capital-output ratios are constant, which might not hold. (ii) The model is criticized for its instability.
8	EM-8	(i) The mathematical technique assumes constant returns to scale and that the factors of production are perfectly substitutable, which is not always the case. (ii) The technique assumes that technology affects productivity uniformly, ignoring the fact that technology may impact labor and capital differently.
9	EM-9	(i) The mathematical technique assumes a stable trade-off between inflation and unemployment. However, the stagflation of the 1970s, where high inflation coexisted with high unemployment, challenged this assumption. (ii) The technique provides a short-term relationship, not holding true in the long run according to the natural rate hypothesis.

Applications of the Models in the Field of Economics

Applications of the considered models from Economics perspectives are listed as-

- EM-1: The model is used in econometrics to quantify the relationship between variables. e.g., it is used to model how changes in the price of a product affect the quantity of the product sold. This allows economists to predict future sales, understand trends, and make strategic decisions.
- EM-2: Economists use the Taylor series to approximate functions that cannot be easily computed. e.g., the Taylor series can be used to approximate a firm's production function in terms of labor and capital.
- EM-3: In economics, the model is used to describe the growth of population, inflation, and investments over time under the assumption of a constant growth rate. e.g., it is used to project future population growth for a given constant growth rate.

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- EM-4: The technique is fundamental in finance, a branch of economics. It is used to calculate the future value of investments for a given particular interest rate. This helps to understand the investments' growth over time and thus helps in decision-making regarding savings, investments, loans, and pensions.
- EM-5: It is used in economics to model situations where growth is initially exponential but then slows down as the capacity limit is reached, such as market saturation for a particular product or service. This is used for forecasting demand and strategizing about market entry and expansion.
- EM-6: The model is used in development economics to understand how economies grow over the long run. Policymakers use this model to inform decisions on investment, savings, and technological development policies.
- EM-7: The model is used to analyze economic dynamics and to calculate the amount of investment required to achieve a certain level of growth.
- EM-8: The function is used in labor economics and macroeconomics to understand the impact of varying levels of inputs on output, and to understand the concept of returns to scale.
- EM-9: The technique is used by policymakers to manage macroeconomic policy, particularly in the context of the trade-off between unemployment and inflation.

Comparative Roles of the Models in Analysing Economic Phenomena

The comparative roles of the mathematical models in predicting diverse economic phenomena are enlisted as-

EM-1 vs EM-2: EM-1, a simple predictive tool, uses past data trends to forecast future economic variables. In contrast, EM-2 provides a more detailed approximation for complex economic functions, capturing small changes effectively.

EM-3 vs EM-4: The EM-3 model is critical in predicting unbounded growth, such as GDP over time under certain conditions whereas the EM-4 Formula is applied to finance, predicting the future value of investments and loans.

EM-5 vs EM-6 and EM-7: The EM-5 model predicts constrained growth, such as market saturation, while the EM-6 and EM-7 models are fundamental in predicting long-term economic growth, incorporating variables such as labor, capital, and technological progress.

EM-8 vs EM-9: EM-8 predicts output given inputs of labor and capital, reflecting diminishing returns. On the other hand, the EM-9 model predicts the trade-off between inflation and unemployment, instrumental in fiscal policy decisions.

Advanced Mathematical Expressions for First Five Models

The advanced mathematical expressions EM_A-j ($1 \leq j \leq 9$) for the models representing the general economic case can be expressed as-

EM_A-1: In the case of multiple linear regression, where there are multiple independent variables (McKelvey and Zavoina, 1975), the EM-1 expression takes the form,

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$D_V = A_o + A_1 I_{V1} + A_2 I_{V2} + ... + A_n I_{Vn} + e_r$

Here, A_i represents the effect of the i^{th} independent variable I_{Vi} on the dependent variable D_V, A_o is the D_V-intercept, and e_r is the error term.

EM_A-2: For a function of multiple variables, the EM-2 expansion becomes more complex (Wright, 1940). For a function G(s, t) around the point (s_o, t_o), the expansion is,

$G(s, t) = G(s_o, t_o) + G_s(s_o, t_o)(s-s_o) + G_t(s_o, t_o)(t-t_o) + ...$

Here, G_s and G_t represent the partial derivatives w.r.t. s and t respectively.

EM_A-3: When the growth rate varies with time, the EM-3 formula becomes a differential equation (Zhang, 2005),

$dQ/dt = R_g(t) Q$

Here, $R_g(t)$ is a function expressing how the rate of growth changes over time.

EM_A-4: For continuously compounded interest, the EM-4 formula becomes (Biggs, 2013),

$M_A = P_A exp(R_A t)$

EM_A-5: The EM-5 model can be extended to include a harvesting or death term (Dennis, 1989), giving the form,

$dP/dt = I P (1 - P/C_c) - D$

Here, D represents the number of individuals removed from the population per unit of time.

The advanced models for the rest four models take a more complex form and hence is a matter to deal with in a separate dedicated study.

Potential Future Research Directions

EM-1 technique can often fail to account for complex, nonlinear relationships and interactions between variables. Future research could focus on integrating machine learning techniques to capture such patterns or creating hybrid models that can account for both linear and nonlinear interactions. EM-2 approximations are local in nature and may not represent global economic phenomena accurately. More research is needed on developing methods to better estimate the error associated with using them for global predictions. The EM-3 model assumes a constant growth rate, while real-world economies have cycles of booms and busts. Future research could look into incorporating elements of cyclical and structural change in growth patterns into these models. The EM-4 formula assumes a constant interest rate that does not hold over long periods. It can be modified by incorporating variable interest rates or by modeling scenarios under different interest rate policies. The EM-5 model assumes that growth will eventually slow and stop, which may not apply to all economic phenomena. It is required to develop and integrate models that account for different growth dynamics. For the EM-6 model, researchers could look at how to incorporate varying savings rates, technology advancements, and impacts of human capital and inequality. The EM-7 model can be modified to incorporate the complex impacts of investment, like how it is used, how efficiently it is converted into output, and its

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distribution. The EM-8 model assumes that factors of production are paid for their marginal products, which is a strong assumption. Modification in the model is required to incorporate different pricing mechanisms, market imperfections, and dynamics of labor and capital markets. The EM-9 model's assumption of an inverse relationship between inflation and unemployment needs to be modified. Researchers could investigate different forms of the curve, the impact of factors like technology and globalization, and potential alternatives to the model.
Future research would benefit from a multi-model approach, utilizing the strengths of each model where applicable, and understanding that no one model can universally apply to every situation. Innovation in computational techniques and data collection may also open new opportunities to improve these models and better understand economic dynamics.

Conclusion

The paper examined nine mathematical models and their application to economic phenomena. The thorough analysis of available literature revealed that each model, whether Linear Regression, Taylor Series, or others, provides unique insights but also has inherent limitations. The advanced mathematical expressions given for the five models make it easy to handle more realistic economic phenomena. Understanding these strengths and weaknesses helps to establish their contextual applicability and forms the groundwork for a more advanced approach to economic analysis. The models should be chosen and interpreted based on the specific economic phenomena under study. The paper encourages future research to explore hybrid and multi-model strategies to address the complexities of real-world economic scenarios. Moreover, advancements in mathematical techniques offer possibilities to refine these models further. This exploration serves as a stepping stone towards more comprehensive, inclusive, and adaptive economic modeling in the future.

Statements and Declarations

Funding

This research did not receive any grant from funding agencies in the public, commercial, or not-for-profit sectors.

Conflict of Interest

The authors declare that they have no conflict of interest.

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