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A Study on Theoretical Discrete Distributions

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Abstract: This paper deals with the study of the well known theoretical discrete distributions like Binomial, Poisson, Geometric and Hyper Geometric distributions. This paper enhances the academic knowledge of the students in studying the properties, applications and importance of the theoretical discrete distributions.

Key words: Probability distribution, Random Variable, Mathematical expectation, Binomial distribution, Poisson distribution.

Introduction: The probability distributions are most prominent in statistical theory and application. The statistical measures like averages, dispersion, skewness, kurtosis, etc., for the frequency distributions, not only give us the nature and form of the sample data but also help us to formulate certain ideas about the population characteristics. But, by the theoretical distributions we can study the characteristics of the population in a more scientific manner and draw the inferences effectively. Generally, in population the values of the variable may be distributed according to some definite probability law and the corresponding probability distribution is known as theoretical probability distribution. Binomial, Poisson are the most popular discrete probability distributions which deals with the discrete random variables of the experiment.

Random variable: In most experiments, we may be interested in knowing which of the outcomes has occurred in numbers. For example, when a coin is tossed n times simultaneously, we may be interested in knowing the occurring of number of tails and when two dice are thrown, one may be interested in information about the sum of points on them. Thus, we associate a real number with the outcome of the experiment. In other words, a function whose domain is the set of all possible outcomes and whose range is a subset of the set of real values. Such a function is known as a random variable.

Random variable means a real number connected with the outcome of the random experiment. In other words, a random variable is a variable whose value is determined by the outcome of a random procedure. If the outcome of the experiment takes countable number of values then it is known as discrete random variable, the example is number of heads in three tosses of a coin,

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number of class rooms in a building, number of petals in a rose flower. If the outcome of the experiment takes any possible value between certain range it is known as continuous random variable, the example is marks of students, height of a person, life of an electric bulb. The probability distributions related to discrete and continuous random variables are known as discrete and continuous probability distributions.

Consider a random experiment E, which consists of two tosses of fair coin and the random variable which is the number of heads X (0,1 or 2)

Outcome	HH	HT	TH	TT
Value of X	2	1	1	0

Mathematical expectation: Once we have constructed the probability distribution for a random variable, the next step is to determine the mean, variance and other characteristics of the variable. The mathematical expectation of the discrete random variable X with probability mass function f(x) is given by

$$E(X) = \sum_x x\, f(x) \text{ (for discrete r.v.)}$$

The mathematical expression for computing the expected value of a continuous random variable X with the probability density function f(x) is given by

$$E(X) = \int_{-\infty}^{+\infty} x\, f(x) dx \text{ (for continuous r.v.)}$$

By making use of mathematical expectation, we can determine the mean, variance, 3^{rd} and 4^{th} order moments of the probability distribution and by which we can study the various characteristics of the random variable.

Binomial Dsitribution

The Binomial distribution is an important discrete probability distribution in probability theory as from this distributions we can generate most of the discrete and continuous probability distributions. The distribution was first proposed by James Bernoulli in 1700 to deal with random experiments having two possible outcomes namely success and failure.

Consider a random experiment with two possible outcomes only-one is called success and other is called failure. Let the probability of success be p and that of failure is 1-p=q(say). Such an experiment is called a Bernoulli experiment.

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Suppose we repeat a Bernoulli experiment n times. Let X denote the number of times the success occurs. Clearly X can take any of the values 0, 1, 2,..........,n.

Now, P[X=x]=P[getting x success and (n-x) failures in the n repetitions of the Bernoulli experiment] = $n_{c_x}\ p^x q^{(n-x)}$

This distribution is called the Binomial distribution.

a) Definition: A discrete random variable X is said to follow the binomial distribution with parameters 'n' and 'p' if its probability density function is given by,

$$f(x) = {}^nC_X\ p^x\ q^{n-x}, \qquad x= 0, 1, 2, \quad , n \qquad 0<p<1\ ;\ p+q=1$$

The distribution is usually denoted by the symbol B(x, n, p) or B(n,p)
The various probabilities of the Binomial random variable are ${}^nC_0\ p^0\ q^{n-0}$, ${}^nC_1\ p^1\ q^{n-1}$, ${}^nC_2\ p^2\ q^{n-2}$, ... ${}^nC_r\ p^r\ q^{n-r}$, ... ${}^nC_n\ p^n\ q^{n-n}$.

We can observe that the above probabilities are the terms of the binomial expansion of $(q+p)^n$, so the distribution is rightly termed as binomial distribution.

b) Assumptions: The binomial distribution can be used under the following conditions.

1) The random experiment is repeated a finite number of times. In other words n, the number of trials must be fixed and finite .
2) The outcome of each trial may be classified into two mutually exclusive categories, known as success (happening of event) and failure (non –happening of event).
3) All the trials are independent of each other.
4) The probability of success, p in each trial is constant.

c) Properties: The important characteristics or basic properties of the binomial distribution are

1) The binomial distribution is a discrete probability distribution in which the random variable assumes finite number of countable values.
2) The distribution has n (number of trails) and p (probability of success) as parameters.

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3) The distribution will be symmetrical if p and q=1-p are equal otherwise asymmetrical.
4) The binomial distribution may be represented graphically, taking number of successes on horizontal axis and the probabilities or frequencies on vertical axis.
5) The mean and variance of Binomial distribution are np and npq.
6) First four central moments of the distribution are estimated as,
    i. $\mu 1 = 0$
    ii. $\mu 2 = npq$
    iii. $\mu 3 = npq(q-p)$
    iv. $\mu 4 = 3n2p2q2 + npq(1-6pq)$
7) Measure of Skewness $\beta_1 = (q-p)^2/ npq$ and the distribution is positively skewed.
8) Measure of kurtosis, $\beta_2 = 3+ (1-6pq)/npq$
9) It has one or two modal values. When (n+1)p is an integer there are two modes. They are, (n+1)p and {(n+1)p}-1. When (n+1)p is not an integer, mode is the integral part of (n+1)p
10) Moment Generating Function of the Binomial distribution is $(q+pe^t)^n$
11) The binomial distribution possesses additive property, in other words if X and Y are two independent binomial variables with parameters (n_1,p) and (n_2, p) then, their addition i.e., X+Y is also binomial variable with parameters (n_1+n_2, p).

d) Applications: The important applications of binomial distribution are

1) Binomial distribution is often used in Quality Control for estimating probabilities of defective items.
2) Used in Radar detection.
3) Used in estimation of Reliability of systems.
4) Used to estimate the number of rounds fire from gun hitting a target.
5) Used in problems related to disease of people work in industry.

Poisson Distribution

The Poisson distribution is an important discrete probability distribution used for modelling natural phenomenon. This distribution was first proposed by the French Mathematician Simon Denis Poisson in 1837. We may come across the cases where the number of times an event in a trail occurs is indefinitely very large. In such cases it is not possible to obtain the theoretical probabilities and frequencies using binomial distribution, as the n is not known. In such cases, poisson distribution

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many be conveniently used to get the theoretical probabilities and frequencies.

a) **Definition:** A discrete random variable X is said to follow the Poisson distribution with parameter λ if its probability mass function (p. m. f) is given by

$$f(x) = (e^{-\lambda} \lambda^{x}) / x! \qquad \text{where } x = 0,1,2,3,\dots\dots\dots \text{ and } \lambda>0$$

This distribution is usually denoted by the symbol P (x, λ) or simply P(λ).

The poisson distribution is a limiting form of the binomial distribution under the following conditions:

i) n, the number of trials are indefinitely very large.
ii) P, probability of success in each trial is constant.
iii) np = λ (say), is finite.

b) **Properties:** The poisson distribution has the following important properties.

1) The variable is discrete.
2) The event can only be either a success or failure.
3) The number of trials n, is finite and large.
4) The probability of success is very small and probability of failure is almost equal to unity.
5) The mean and variance of poisson distribution are equal and is λ.
6) First four central moments of the distribution are estimated as,
   i. $\mu 1 = 0$
   ii. $\mu 2 = \lambda$
   iii. $\mu 3 = \lambda$
   iv. $\mu 4 = 3\lambda+\lambda$
7) Measure of skewness $\beta_1 = 1/\lambda$, since $\beta_1>0$, the poisson distribution is positively skewed.
8) Measure of Kurtosis $\beta_2 = 3+ (1/\lambda)$, since $\beta_2 > 3$, the poisson distribution curve is leptokurtic.
9) If λ is an integer value, there are two modes for the Poisson distribution- λ and λ-1. Also if λ is not an integer, there is only a single mode for the Poisson distribution i. e. the integral part of λ.

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10) The poisson distribution possesses additive property, i.e., if X and Y are two independent poisson variables with λ_1 and λ_2 as parameters then, their sum X+Y is also a poisson variable with $\lambda_1+\lambda_2$.

c) Applications

Generally, the Poisson distribution is used to describe the number of occurrences of a rare event in a short period. Some examples are given below.

1) To count the number of wrong telephone calls receiving in a house in a particular day.
2) To count the dimensional errors in engineering drawing.
3) To count the number of suicides reported in a municipal area in a week.
4) To count the number of printing mistakes at each page of the book.
5) To count the number of accidents at a traffic junction in a particular city.
6) To count the number of number of defects in a bale of cloth.
7) To count the number of bacteria per unit.
8) To count the number of customers arriving at a service facility.

Conclusion

This paper is presented as an overview of the discrete distributions namely, binomial and poisson.

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