

Individual And Collective Pricing Strategies For A Multi-Layer, Multi-Channel Supply Chain With Suggested Selling Price By Manufacturer

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Abstract

This article analyses a pricing policy and coordination among members of a three-layer, multi-channel and multi-echelon supply chain, consisting of manufacturers, distributors and retailers. The demand is considered a linear declining function of time and selling price at the retailer's end for a single item. The maximum profit function per unit of time is earned for each supply chain member subjected to the expenditures. In urban areas, holding cost of goods is more expensive than rural areas, so a holding cost-sharing concept among the distributors and retailers is considered in this article. In this study we have maximized the initial lot size, selling price and replenishment time for retailers, initial lot size, and wholesale price for distributors and manufacturers. This study is analysed in two frameworks first one is decentralized, and other one is centralized. The optimality conditions of each supply chain member's profit function have been derived with respect to the decision variables and propositions. The results are shown in the data table to illustrate the model. We have also done sensitivity analysis with numerical examples.

Key Words and Phrases: Selling Price, Holding Cost, Multi-Layer Multi-Channel, Supply Chain, Individual, Collective Strategies.

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1. Introduction

Globalization has developed the complexity of coordination among supply chain members, and recently from the past one-decade globalization has increased exponentially, consequently, the supply chain sector faces major problems. Due to a lack of co-ordination, the needs of consumers are not satisfied over time and therefore the earnings of supply chain members are affected. To tackle these types of problems, increase earnings, and fulfill the customers' needs within time, many models/articles have been designed by many authors and researchers of supply chain management in this research area. In 1994 Parlar and Weng[18] developed a supply chain inventory model for a single supplier and single retailer under the situation of manufacturer's Stackelberg in

which they considered a concept of quality discount scheme. Further, the model of Parlar and Weng[18] is extended by Weng[26] which comprised a single supplier and multi distributors under the situation of increasing quantity discount policies with lot size. For the development of an inventory model, a relationship is required between price and demand and a very often a convenient price-demand relationship function is chosen arbitrary, but Lau and Lau[2] and Gunasekaran & Kobu [3] suggested a model in which they used different price-demand relationship curve's shape and studied the effects on model output.

Recently, it has been observed that most of the researcher's study two-layer supply chain problems but in real life supply chain networks are more complex and each stage has more supply chain members. Daya et al.[17] formulated a model for a three-layer supply chain which consisted a one manufacturer, one supplier and multi-retailers. Chan et al.[9] optimized the production rate for exponential deteriorating single item for an integrated two-layer supply chain model, consisting of a single vendor and a retailer. This study is an advanced mathematical model compared to traditional by considering variable production rate. Mehata et al.[21] formulated dynamic decision strategies for deteriorating items under price inflation and permissible payment delays. In this study, he adopted an iso-elastic and selling price-dependent demand and optimized retail price replenishment time and finite time horizon. Hariom et al.[15] suggested an inventory model for time-dependent linear demand under three levels of production system considering shortage. Pal et al. [8], Chang et al. [10] and Fang et al. [20] have suggested an optimal decision policy for a dual supply chain in a competitive environment considering green supply chain strategies and promotional efforts.

Lin et al.[13] developed an inventory model for a deteriorating item which is deteriorate in quality and quantity. This study is designed for two stage trade credit policy and optimized the retailers responses when involving both quality and quantity losses. Singh et al.[14] designed an inventory model of an supply chain for deteriorating item under selling price. This study optimized retailer's replenishment rate when demand rate is declining with time. Cardenas- Barron and Sana[16] formulated two two-layer supply chain inventory model for a promotional efforts cost sensitive demand incorporating a delay of payment is offered by supplier to the retailer. Pal et al.[7] formulated the optimal lot size formulas for supplier and production rate for manufacturers under three stage trade credit financing policy for three layer supply chain which consisted supplier, manufacturer and retailers. In this study they assumed that, sequentially provides a fixed credit

period by supplier to the manufacturer and manufacturer provides to the retailer and retailers provide, to the customers.

Zhao and Chen[24] analyzed the pricing strategies for a two-echelon supply chain inventory model which consisted a single manufacturer and two retailers. Song and He et al.[27] Designed a three-layer supply inventory model for products of the agricultural sector. The study is developed for two different strategies which one are centralized and decentralized and optimized unit online selling price, unit logistics distribution price, fresh-keeping effort. Giri et al.[6] investigated a co-ordination issue in a three-layer supply chain with single raw material supplier and single manufacturer and single retailer under the demand is not deterministic. The retailing price, sales effort, and order quantity are determined and optimized at retailers ends. The profit functions of both supply chain member are optimized under two different strategies, decentralized and centralized. Singh et al.[12] Suggested a co-ordination policy for a production system considering finished products and raw materials under different situations. Xu et al. [25] suggested a supply chain coordination policy for an online platform under the green channel supply chain technology.

A three-layer multi-channel and multi-echelon supply chain inventory model is formulated by Modak et al.[19] for a single item. A reverse supply chain coordination policy for multi -collector, multi- distributors and single manufacturers is developed by Nigwal et al.[4]. Shaikh et al.[1] developed two level trade credit policy considering with expiration rate and impact of in their demand under nonzero inventory and partial backlogged. Singh et al. [5] have explained supply chain model, In this study they assumed that the supply chain consisted of one manufacturer, multi retailers and distributors as the supply chain members. Seasonal products deteriorate very fast and after sales season these become useless and the deterioration rate is controlled by item preservation technology. Using this concept Khedlekar et al.[24] suggested an inventory model for pricing strategies, under the declining demand of an item by using preservation technology.

Nowadays is it need to reuse the product is going felt, on the basis of this concept Shah et al. [23] developed a closed-loop supply chain inventory model which consisted of one manufacturer and one retailer. In this article, they assumed the manufacturer and retailer both optimize their own profit by product retailing and recycling and play a social responsibility. The study is analysed in two different frameworks first one is centralized and second one is decentralized. Panda et al.[22]

suggested a three-layer echelon supply chain model which consisted a single manufacturer, multi-distributor and multi-retailer. In this study a systematic co-ordination strategy is formed and benefit benefit-sharing contract is made for all supply chain members for deteriorating single products. In the production system disruption is a common phenomenon in real life. Shukla and Khedlekar[11] designed an inventory model for convertible items in which they consider the item that converts one form to more than one another forms by investing conversion cost and time. They optimized the total convertible cost and conversion time for deteriorating products by assuming deterioration rates differ at each convertible stage.

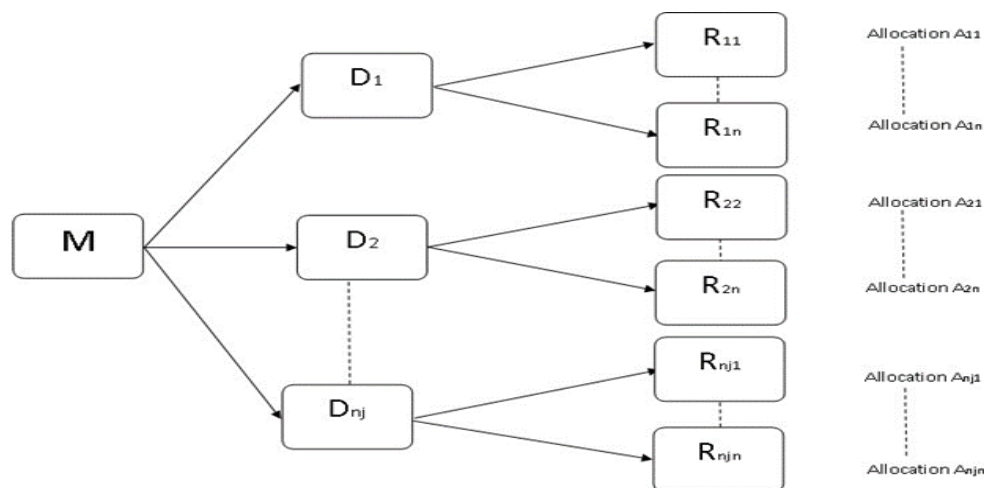


Figure 1: Supply Chain Distribution Network

In this article, we consider a three-layer multi-channel and multi-echelon supply chain in which a single manufacturer occupies the first echelon stage, second stages are occupied by multi distributors and third stages are also occupied by multi-retailers (Fig.1). Initially, the manufacturer provides a fixed lot of a single item to the j^{th} ($j = 1, 2, \dots, n$) distributors and j^{th} distributors also provide a single item to the i^{th} ($i = 1, 2, \dots, n_j$), ($j = 1, 2, \dots, n$) retailers, where each retailer is associated with to a certain distributor. The total demand all retailers of single item is fulfilled by all distributors and the total demand of all distributors' is fulfilled by the single manufacturer. Manufacturers and distributors obey the EOQ delivery policy. The finite replenishment time for retailers must be equally applicable for all distributors as well as manufacturers, therefore we will find replenishment time only for retailers.

This research aims to find optimal replenishment time, selling price, and initial lot size for retailers in a centralized and decentralized framework under the demand of retailer's end-suggested selling

price by the manufacturer of the item. The demand is a linearly declining function of time, and selling price. Furthermore, it has been also assumed that a holding cost is shared among the retailers and distributors.

2 Notations and Assumptions

The following notations are used in this model.

- (i) s^m : Maximum selling price per unit item suggested by the manufacturer.
- (ii) d_{ij}^R : ij^{th} retailer's demand per unit time per unit of the item.
- (iii) d_j^D : j^{th} distributor's demand per unit time per unit of the item.
- (iv) d^M : Manufacture's demand per unit time per unit item.
- (v) s_{ij}^R : Selling price per unit item for ij^{th} retailer in a decentralized framework, where $s_{ij}^R > W_j^d$.
- (vi) s_{ij}^{RC} : Selling price per unit item for ij^{th} retailer in centralized framework.
- (vii) w_j^d : Distributor's wholesale price per unit item, where $w_j^d > w^m$.
- (viii) w^m : Manufacturer's wholesale price per unit item, where $w^m > c$.
- (ix) c : Production cost per unit item for manufacturer.
- (x) z_{ij}^R : ij^{th} retailer's average profit in a decentralized framework.
- (xi) z_j^D : j^{th} distributor's average profit in decentralized framework.
- (xii) z^M : Manufacturer's average profit in a decentralized framework.
- (xiii) n : Number of distributors.
- (xiv) n_j : Number of retailers.
- (xv) z^c : Average profit of the whole channel in a centralized framework.
- (xvi) β : Difference coefficient of s_{ij}^R and s^m , when either $s_{ij}^R \geq s^m$ or $s_{ij}^R \leq s^m$.
- (xvii) α : Price sensitive parameter of demand function.
- (xviii) T : Finite time horizon.
- (xix) $q_{ij}^r(t)$: Initial lot size of ij^{th} retailer's end.
- (xx) $q_j^d(t)$: Initial lot size of j^{th} distributor's end.
- (xxi) $q^m(t)$: Initial lot size of manufacturer.
- (xxii) δ : Holding cost sharing coefficient.
- (xxiii) h : Holding cost per unit item per unit time.

Assumptions

The following assumptions are made for this model

- (i) Demand per unit time of the item in the market is $d_{ij}^R = a_{ij} - b_{ij}t - \alpha s_{ij}^R + \beta(s^m - s_{ij}^R)$, is a linear declining function of t , selling price and difference coefficient of suggested selling price and selling price, where a_{ij} is initial demand scale parameter, β is difference coefficient of s^m and s_{ij}^R , $a_{ij} > 0, b_{ij} > 0, \beta > 0, \alpha > 0$, and $0 \leq t \leq T$.
- (ii) Holding costs are constant and shared among distributors and retailers.
- (iii) $a_j = \sum_{i=1}^{n_j} a_{ij} b_j = \sum_{i=1}^{n_j} b_{ij}$ and $a = \sum_{i=1}^{n_j} \sum_{j=1}^n a_{ij}, b = \sum_{i=1}^{n_j} \sum_{j=1}^n b_{ij}$
- (iv) There is no competitive environment among second and third echelon stages.
- (v) Finite interval time T is evaluated only for retailers and which is equally applicable on whole supply chain.

3. Individual Pricing Strategy

In this framework the manufacturer acts as a Stackelberg leader, distributors and retailers act as followers of manufacturer. Although, the manufacturer is the Stackelberg leader of supply chain, but he cannot determine selling price of the item, he may only suggest a selling price publicly, at which the item is expected to be sold. In common parlance, such selling price is called maximum suggested selling price (mssp). Because all channel members are located sequentially in the different echelon stage, they can independently decide to optimize their individual goals. As per wholesale price, demand of the item and based on known information, retailers' may take own strategic decision about his goal. Therefore, the retailers' model could be formulated first as follows.

3.1. Retailers' Individual Pricing Strategy

Manufacturer knows well about the specification of their item and related manufacturing expenditure. Therefore, manufacturer determines the maximum selling price at which the item expected to be sold. The maximum suggested selling price is generally printed on packet or tag of the item. Generally, as per the market situations and quality of item, consumers are either satisfied or dissatisfied with manufacturer's determined retail price. Let a manufacturer provides stock of the item to the n distributors $D_{1j}, D_{2j}, D_{3j}, \dots, D_{nj}$. Distributors also provide lots of the item to the $R_{11}, R_{12}, R_{13}, \dots, R_{nn}$ retailers respectively. Let ij^{th} retailer receives the stock, at time $t, t \in [0, T]$.

The rate of change in the ij^{th} retailer's initial lot size q_{ij}^R units of item of ij^{th} retailer. At any time t following nonlinear equation represents the inventory status for ij^{th} retailer

$$\frac{dI_{ij}^R(t)}{dt} = -d_{ij}^R, \text{ where } 0 \leq t \leq T \quad (1)$$

with initial condition $I_{ij}^R(t) = 0$, at $t = T$, where $i = 1,2,3, \dots, n_j$ and $j = 1,2,3, \dots, n$, Equation (1) yields

$$I_{ij}^R(t) = a_{ij}(T - t) - \frac{b_{ij}(T^2 - t^2)}{2} + (\alpha + \beta)s_{ij}^R(t - T) + \beta s^m(T - t) \quad (2)$$

The initial inventory level $I_{ij}^R(0) = q_{ij}^R$ for ij^{th} retailer's end at finite time $t = 0$, where $t \in [0, T]$ will be

$$I_{ij}^R(0) = q_{ij}^R = a_{ij}T - \frac{b_{ij}T^2}{2} - (\alpha + \beta)s_{ij}^RT + \beta s^mT \quad (3)$$

The total sales revenue SR_{ij}^R in the replenishment time period $[0, T]$ could be formulated as

$$SR_{ij}^R = \int_0^T s_{ij}^R q_{ij}^R dt \quad (4)$$

$$SR_{ij}^R = s_{ij}^R \left(a_{ij}T - \frac{b_{ij}T^2}{2} - (\alpha + \beta)s_{ij}^RT + \beta s^mT \right)$$

Purchase expenditure PE_{ij}^R of ij^{th} retailer is

$$PE_{ij}^R = \int_0^T w_j^d q_{ij}^R dt \quad (5)$$

$$PE_{ij}^R = w_j^d \left(a_{ij}T - \frac{b_{ij}T^2}{2} - (\alpha + \beta)s_{ij}^RT + \beta s^mT \right)$$

The inventory holding expenditure IHE_{ij}^R of ij^{th} retailer is

$$IHE_{ij}^R = h \int_0^T I_{ij}^R(t) dt \quad (6)$$

$$IHE_{ij}^R = h \left(\frac{a_{ij}T^2}{2} - \frac{b_{ij}T^3}{3} - (\alpha + \beta)s_{ij}^R \frac{T^2}{2} + \beta s^m \frac{T^2}{2} \right)$$

Hence, the average profit function per unit time is

$$z_{ij}^R = \left[(s_{ij}^R - w_j^d) \left\{ a_{ij} - \frac{b_{ij}T}{2} - (\alpha + \beta)s_{ij}^R + \beta s^m \right\} \right. \\ \left. - h\delta \left(\frac{a_{ij}T}{2} - \frac{b_{ij}T^2}{3} - (\alpha + \beta)s_{ij}^R \frac{T}{2} + \beta s^m \frac{T}{2} \right) \right] \quad (7)$$

Proposition 3.1 If the demand of items is uniformly at all retailer's end with respect to time T and $s_{ij}^R > w_j^d$ ($i = 1,2,3 \dots m$ and $j = 1,2,3 \dots n$). Then the ij^{th} retailer's profit function shows concavity in selling price s_{ij}^R and replenishment time T , if $\frac{4}{3}(\alpha + \beta)h\delta b_{ij} - \frac{1}{4}(\delta h(\alpha + \beta) - b_{ij})^2 > 0$.

Proof: The second order partial derivative of ij^{th} retailer's profit function z_{ij}^R with respect to s_{ij}^R and T respectively are

$$\begin{aligned}\frac{\partial^2 z_{ij}^R}{\partial s_{ij}^{R2}} &= -2(\alpha + \beta) \\ \frac{\partial^2 z_{ij}^R}{\partial s_{ij}^R \partial T} &= \frac{h\delta(\alpha + \beta) - b_{ij}}{2} \\ \frac{\partial^2 z_{ij}^R}{\partial T^2} &= -\frac{2b_{ij} h\delta}{3}\end{aligned}$$

Retailer's profit function z_{ij}^R shows concavity with respect to s_{ij}^R and T , if the Hessian matrix of z_{ij}^R , is negative semi definite

$$HM = \begin{bmatrix} \frac{\partial^2 z_{ij}^R}{\partial s_{ij}^{R2}} & \frac{\partial^2 z_{ij}^R}{\partial s_{ij}^R \partial T} \\ \frac{\partial^2 z_{ij}^R}{\partial s_{ij}^R \partial T} & \frac{\partial^2 z_{ij}^R}{\partial T^2} \end{bmatrix} = \begin{bmatrix} -2(\alpha + \beta) & \frac{h\delta(\alpha + \beta) - b_{ij}}{2} \\ \frac{h\delta(\alpha + \beta) - b_{ij}}{2} & -\frac{2b_{ij} h\delta}{3} \end{bmatrix} \quad (8)$$

Expansion of Hessian matrix gives $16(\alpha + \beta)b_{ij} h\delta - 3(\delta h(\alpha + \beta) - b_{ij})^2 > 0$, if $\beta > 0, \alpha > 0$ and hence Hessian matrix of z_{ij}^R , is negative semi definite in s_{ij}^R and T if $16(\alpha + \beta)b_{ij} h\delta - 3(\delta h(\alpha + \beta) - b_{ij})^2 > 0$.

Proposition 3.2 If the demand is uniformly decreasing function of suggested price and time at all ij^{th} retailer's end and $s_{ij}^R > w_j^d$ ($i = 1,2,3 \dots m$ and $j = 1,2,3 \dots n$). Then optimum selling price s_{ij}^R is given by the equation (9) and optimum replenishment time T can be find by satisfying the equation (10).

$$s_{ij}^R = \frac{w_j^d}{2} + \frac{\beta s^m}{2(\alpha + \beta)} + \frac{\delta h T}{4} + \frac{2a_{ij} - b_{ij} T}{4(\alpha + \beta)} \quad (9)$$

$$(s_{ij}^R - w_j^d)b_{ij} + \delta h(a_{ij} - (\alpha + \beta)b_{ij} + \beta s^m) - \frac{4}{3}b_{ij} \delta h T = 0. \quad (10)$$

Proof: Equate to zero the first order partial derivatives of equation (7), yields

$$(\alpha + \beta)(s_{ij}^R - w_j^d) - \frac{2a_{ij} - b_{ij} T}{2} + (\alpha + \beta)s_{ij}^R - \beta s^m + h\delta(\alpha + \beta)\frac{T}{2} = 0 \quad (11)$$

$$\frac{b_{ij}(s_{ij}^R - w_j^d)}{2} + \delta h \left[\frac{a_{ij}}{2} - \frac{2b_{ij} T}{3} - \frac{(\alpha + \beta)s_{ij}^R}{2} + \frac{\beta s^m}{2} \right] = 0 \quad (12)$$

Solution of these simultaneous equation gives the required results.

3.2. Distributors' Individual Pricing Strategy

Let $D_{1j}, D_{2j}, D_{3j}, \dots, D_{nj}$ be the n^{th} distributors and the demand of distributors end is a sum of all respective retailer's demand.

Corollary 3.1 If the coefficient b_{ij} is uniformly distributive with respect to time and $a_j = \sum_{i=1}^{n_j} a_{ij} b_j = \sum_{i=1}^{n_j} b_{ij}$, then the demand of items at d_j^D distributors' end can be determined by the following formula

$$d_j^D = \sum_{i=1}^{n_j} d_{ij}^R = a_j - b_j t - (\alpha + \beta) \sum_{j=1}^{n_j} s_{ij}^R + n_j \beta s^m \quad (13)$$

Now at the time t the rate of changes in the j^{th} distributor's inventory level can be balanced by the sum of all associated retailers' demand which are affiliated to the j^{th} distributor. Therefore, at any time t , j^{th} distributor's inventory can be represented by the equation

$$\frac{dI_j^D(t)}{dt} = -d_j^D \quad \text{where } 0 \leq t \leq T \quad (14)$$

with initial condition $I_j^D(t) = 0$, at $t = T$, where $j = 1, 2, 3, \dots, n$, Equation (14) yields

$$I_j^D(t) = a_j(T - t) - \frac{b_j}{2}(T^2 - t^2) + (\alpha + \beta) \sum_{j=1}^{n_j} s_{ij}^R(t - T) + \beta s^m n_j(T - t) \quad (15)$$

The initial lot size for j^{th} retailer at any time $t = 0$, where $t \in [0, T]$ will be

$$I_j^D(0) = q_j^D = a_j T - \frac{b_j T^2}{2} - (\alpha + \beta) \sum_{j=1}^{n_j} s_{ij}^R T + \beta s^m n_j T \quad (16)$$

The sales revenue of j^{th} distributor SR_j^D in the finite time interval $[0, T]$ can be find as

$$SR_j^D = \int_0^T w_j^d q_j^D dt$$

$$SR_j^D = w_j^d \left(a_j T - \frac{b_j T^2}{2} - (\alpha + \beta) \sum_{j=1}^{n_j} s_{ij}^R T + \beta s^m T n_j \right) \quad (17)$$

Purchase expenditure of j^{th} distributor is

$$PE_j^D = \int_0^T w^m d_j^D dt = w^m \left(a_j T - \frac{b_j T^2}{2} - (\alpha + \beta) \sum_{j=1}^{n_j} s_{ij}^R T + \beta s^m T n_j \right) \quad (18)$$

The inventory holding expenditure IHC_j^d for j^{th} distributor is

$$IHE_j^D = h \int_0^T I_j^D(t) dt = h \left(\frac{a_j T^2}{2} - \frac{b_j T^3}{6} - (\alpha + \beta) \sum_{i=1}^{n_j} \frac{p_{ij}^r T^2}{2} + \frac{\beta s^m n_j T^2}{2} \right) \quad (19)$$

Hence the average profit function per unit time for j^{th} distributor is

$$Z_j^D = \left[(w_j^d - w^m) \left\{ a_j - \frac{b_j T}{2} - (\alpha + \beta) \sum_{j=1}^{n_j} s_{ij}^R + \beta s^m n_j \right\} \right. \\ \left. - h(1 - \delta) \left(\frac{a_j T}{2} - \frac{b_j T^2}{6} - (\alpha + \beta) \sum_{i=1}^{n_j} \frac{s_{ij}^R T}{2} + \frac{\beta s^m n_j T}{2} \right) \right] \quad (20)$$

Proposition 3.3 If the demand is uniformly decreasing function of suggested price and time T at all j^{th} distributors' end and $w_j^d > w^m (j = 1, 2, 3 \dots n)$. Then the optimum whole sale price w_j^d is given by the equation (21).

$$w_j^d = \left[\frac{w^m}{2} + \frac{2a_j - b_j T}{4(\alpha + \beta)n_j} - \frac{\delta h T}{4} - \frac{\beta s^m}{2(\alpha + \beta)} + \frac{h(1 - \delta)}{2} \right] \quad (21)$$

Proof: Partial differentiation of equation (20) with respect to w_j^d yields

$$\frac{\partial z_j^d}{\partial w_j^d} = \left[\begin{array}{c} -\frac{(w_j^d - w^m)n_j(\alpha + \beta)}{2} + \frac{2a_j - b_j T}{4} - \frac{w_j^d n_j(\alpha + \beta)}{2} \\ -\frac{\beta s^m n_j}{2} - \frac{(\alpha + \beta)h\delta n_j T}{4} + \frac{h(1 - \delta)n_j(\alpha + \beta)}{2} \end{array} \right]$$

At the optimum value of w_j^d , $\frac{\partial z_j^d}{\partial w_j^d} = 0$ i.e.

$$\left[\begin{array}{c} -\frac{(w_j^d - w^m)n_j(\alpha + \beta)}{2} + \frac{2a_j - b_j T}{4} - \frac{w_j^d n_j(\alpha + \beta)}{2} \\ -\frac{\beta s^m n_j}{2} - \frac{(\alpha + \beta)h\delta n_j T}{4} + \frac{h(1 - \delta)n_j(\alpha + \beta)}{2} \end{array} \right] = 0 \quad (22)$$

Solution of equation (22), yields

$$w_j^d = \frac{w^m}{2} + \frac{2a_j - b_j T}{4(\alpha + \beta)n_j} - \frac{\delta h T}{4} - \frac{\beta s^m}{2(\alpha + \beta)} + \frac{h(1 - \delta)}{2} \quad (23)$$

Also, profit function z_j^d shows optimality with respect to w_j^d , because

$$\frac{\partial^2 z_j^d}{\partial w_j^{d2}} = -n_j(\alpha + \beta) \quad (24)$$

for, if $\beta > 0$ and $\alpha > 0$.

3.3. Manufacturer's Individual Pricing Strategy

Manufacturer provides a initial lot size of items to all distributors as per their demands.

Corollary 3.2 If the coefficient b_j is uniformly distributive with respect to T and $a = \sum_{i=1}^{n_j} \sum_{j=1}^n a_{ij}$ $b = \sum_{i=1}^{n_j} \sum_{j=1}^n b_{ij}$, then the demand of items at manufacturer's end can be determined by the following formula

$$d^M = \sum_{j=1}^n d_j^D = a - bt - (\alpha + \beta) \sum_{i=1}^{n_j} \sum_{j=1}^n s_{ij}^R + m\beta s^m \quad (25)$$

The rate of changes in the manufacturer's inventory level is balanced by all j^{th} distributor's demand.

At any movement t manufacturer's inventory level can be represented as

$$\frac{dI^m(t)}{dt} = -d^M, \text{ where } 0 \leq t \leq T \quad (26)$$

with boundary condition $I^m(t) = 0$, at $t = T$. Solution of equation (26) yields

$$I^m(t) = a(T - t) - \frac{b(T^2 - t^2)}{2} + (\alpha + \beta) \sum_{i=1}^{n_j} \sum_{j=1}^n s_{ij}^R (t - T) + \beta s^m m(T - t) \quad (27)$$

The initial lot size for manufacturer at time $t = 0$, where $t \in [0, T]$ is

$$I^m(0) = q^m = aT - \frac{bT^2}{2} - (\alpha + \beta) \sum_{i=1}^{n_j} \sum_{j=1}^n s_{ij}^R T + \beta s^m mT \quad (28)$$

The sales revenue in the finite time $[0, T]$ can be determined as

$$SR^m = \int_0^T w^m d^M dt = w^m \left(aT - \frac{bT^2}{2} - (\alpha + \beta) \sum_{i=1}^{n_j} \sum_{j=1}^n s_{ij}^R T + \beta s^m Tm \right) \quad (29)$$

Manufacturing expenditure for manufacturer is

$$MC^m = c \int_0^T d^M dt = c \left(aT - \frac{bT^2}{2} - (\alpha + \beta) \sum_{i=1}^{n_k} \sum_{j=1}^n s_{ij}^R T + \beta p_m Tm \right) \quad (30)$$

Hence the average profit function z^m per unit time for manufacturer is

$$z^m = (w^m - c) \left[a - \frac{bT}{2} - (\alpha + \beta) \sum_{i=1}^{n_j} \sum_{j=1}^n s_{ij}^R + \beta s^m m \right] \quad (31)$$

Proposition 3.4 If the demand is uniformly decreasing function of suggested price and time T at manufacturer's end and $w^m > c$. Then the optimum whole sale price w^m is given by the equation (32).

$$w^m = \frac{c}{2} + \frac{2a - bT}{4(\alpha + \beta)m} + \frac{3\beta s^m}{2(\alpha + \beta)} - \frac{\delta hT}{4} - \frac{h(1 - \delta)}{2(\alpha + \beta)} \quad (32)$$

Proof: Partial differentiation of equation (31) with respect to w^m yields

$$\frac{\partial z^m}{\partial w^m} = \left[\begin{array}{c} -\frac{(w^m - c)(\alpha + \beta)m}{4} + a - \frac{bT}{2} - \frac{(\alpha + \beta)w^m m}{4} - \frac{2a - bT}{8} \\ -\frac{m\beta s^m}{4} - \frac{\delta(\alpha + \beta)hmT}{8} - \frac{2a - bT}{4} - \frac{(1 - \delta)hm}{4} + m\beta s^m \end{array} \right] \quad (33)$$

At the optimum value of w^m , $\frac{\partial z^m}{\partial w^m} = 0$, i.e.

$$\left[\begin{array}{c} -\frac{(w^m - c)(\alpha + \beta)m}{4} + a - \frac{bT}{2} - \frac{(\alpha + \beta)w^m m}{4} - \frac{2a - bT}{8} \\ -\frac{m\beta s^m}{4} - \frac{\delta(\alpha + \beta)hmT}{8} - \frac{2a - bT}{4} - \frac{(1 - \delta)hm}{4} + m\beta s^m \end{array} \right] = 0 \quad (34)$$

Solution of the equation (34) yields (32) Also profit function z^m shows optimality with respect to w^m , we have

$$\frac{\partial^2 z_j^d}{\partial w^m} = \frac{-n(\alpha + \beta)}{2}$$

for if $\beta > 0$ and $\alpha > 0$.

4. Collective Pricing Strategy

In this pricing strategy, the whole supply chain members work together as a single unit and all the members of supply chain cooperate perfectly to each other to maximize the performance of supply chain. The manufacturer is a leader of whole supply chain as a single decision maker and all decisions are equally applicable to the whole supply chain members. Therefore, for optimization of whole channel's profit he can take all decisions.

4.1. The Model

Let s_{ij}^{RC} is a selling price of ij^{th} retailer, w_j^d is a whole sale price of j^{th} distributor, w^m is a whole sale price of manufacturer, c is the manufacturing cost, IHC_{ij}^r is the holding cost of ij^{th} retailer and IHC_j^d is holding cost of j^{th} distributor, then the profit function is

$$\begin{aligned} z^c &= \sum_{i=1}^m \sum_{j=1}^{m_j} [(s_{ij}^{RC} - w_j^d)d_{ij}^R - \delta(IHC_{ij}^R)] \\ &+ \sum_{i=1}^n [(w_j^d - w^m)d_j^D - (1 - \lambda)IHC_j^D] + (w^m - c)d^m \\ &= \sum_{i=1}^{n_j} \sum_{j=1}^n [(p_{ij}^r - c)d_{ij}^R - (IHC_{ij}^R)] \end{aligned}$$

Hence the average profit function z^c per unit time is

$$z^c = \left[\begin{array}{l} \sum_{i=1}^{n_j} \sum_{j=1}^n (s_{ij}^R - c) \left(a_{ij} - \frac{b_{ij}T}{2} - (\alpha + \beta)s_{ij}^R + \beta s^m \right) \\ - \sum_{i=1}^{m_j} \sum_{j=1}^m h \left(\frac{a_{ij}T}{2} - \frac{b_{ij}T^2}{3} - \frac{(\alpha + \beta)s_{ij}^R T}{2} + \frac{\beta s^m T}{2} \right) \end{array} \right] \quad (35)$$

Proposition 4.1 If the demand of items is uniformly at all retailer's end with respect to time T and $s_{ij}^{RC} > c$ ($i = 1, 2, 3 \dots m$ and $j = 1, 2, 3 \dots n$). Then the whole supply chain profit function shows concavity in selling price s_{ij}^{RC} and replenishment time T , if $-\frac{4}{3}(\alpha + \beta)hbm - \frac{1}{4}(h(\alpha + \beta)m - b)^2 > 0$.

Proof: Using equation (35), the second order partial derivatives in selling s_{jk}^{RC} , and time T of the profit function respectively are

$$\frac{\partial^2 z^c}{\partial s_{ij}^{RC2}} = -2m(\alpha + \beta) \quad (36)$$

$$\frac{\partial^2 z^c}{\partial s_{ij}^{RC} \partial T} = \left(\frac{h(\alpha + \beta)m}{2} - \frac{b}{2} \right) \quad (37)$$

$$\frac{\partial^2 z^c}{\partial T^2} = \frac{2bh}{3} \quad (38)$$

the profit function z^c must be jointly concave with respect to s_{ij}^{RC} and T , if the Hessian matrix of profit function z^c , is negative semi definite

$$HM = \begin{bmatrix} \frac{\partial^2 z^c}{\partial s_{ij}^{RC2}} & \frac{\partial^2 z^c}{\partial p_{ij}^{RC} \partial T} \\ \frac{\partial^2 z^c}{\partial p_{ij}^{RC} \partial T} & \frac{\partial^2 z^c}{\partial T^2} \end{bmatrix} = \begin{bmatrix} \frac{2bh}{3} & \left(\frac{h(\alpha + \beta)m}{2} - \frac{b}{2} \right) \\ \left(\frac{h(\alpha + \beta)m}{2} - \frac{b}{2} \right) & -2m(\alpha + \beta) \end{bmatrix} \quad (39)$$

Hence, if $-\frac{4}{3}(\alpha + \beta)hbm - \frac{1}{4}(h(\alpha + \beta)m - b)^2 > 0.$, then the Hessian matrix of the profit π^c , must be negative semi definite and thus the profit function π^c is jointly concave in p_{ij}^{rc} and T .

Hence proved it.

Proposition 4.2 If the demand is uniformly decreasing function of suggested price and time T at all ij^{th} retailer's end and $s_{ij}^{RC} > c$ ($i = 1,2,3 \dots m$ and $j = 1,2,3 \dots n$). Then optimum selling price s_{ij}^{RC} is given by the equation (40) and optimum replenishment time T can be find by satisfying the equation (41).

$$s_{ij}^{RC} = \frac{c}{2} + \frac{2a_{ij} - b_{ij}T}{4(\alpha + \beta)} + \frac{\beta s^m}{2(\alpha + \beta)} + \frac{hT}{4} \quad (40)$$

and

$$\frac{s_{ij}^{RC} b_{ij}}{2} - \frac{cb_{ij}}{2} + \frac{a_{ij}h}{2} - \frac{2b_{ij}Th}{3} - \frac{(\alpha + \beta)hs_{ij}^{RC}}{2} + \frac{\beta s^m h}{2} = 0 \quad (41)$$

Proof: Equate to zero the first order partial derivatives of equation (35), yields

$$\frac{\partial z^c}{\partial s_{ij}^{RC}} = \left[\sum_{i=1}^{n_j} \sum_{j=1}^n (s_{ij}^R - c) \left(a_{ij} - \frac{b_{ij}T}{2} - (\alpha + \beta)s_{ij}^R + \beta s^m \right) \right. \\ \left. \{ -(\alpha + \beta) \} + \sum_{i=1}^{n_j} \sum_{j=1}^n h(\alpha + \beta) \frac{T}{2} \right] \quad (42)$$

$$\frac{\partial z^c}{\partial T} = \left[\sum_{i=1}^{n_j} \sum_{j=1}^n (s_{ij}^R - c) \left(-\frac{b_{ij}}{2}\right) - \sum_{i=1}^{m_j} \sum_{j=1}^n h \left(\frac{a_{ij}}{2} - \frac{2b_{ij}T}{3} - \frac{(\alpha+\beta)s_{ij}^R}{2} + \frac{\beta s^m}{2}\right) \right] = 0 \quad (43)$$

Solution of these simultaneous gives the required results.

5. Numerical Example

For illustration of this model, we considered a three-layer echelon supply chain which consists a single manufacturer M, two distributors (D_1, D_2) and four retailers (R_{11}, R_{12}, R_{21} and R_{22}) respectively at each echelon stages. As per Fig.1, each retailer is associated with particular distributor. We consider the following data set for individual and collective pricing strategies, the demand scale parameters are $a_{11} = 1588, a_{12}=1585, a_{21} = 1590, a_{22} = 1581, b_{11} = 0:01, b_{12} = 0.01, b_{21} = 0.01, b_{22} = 0.01$ units, suggested selling price $s^m = 1025$, price sensitive $\alpha = 0.9, \delta = 0.01, h = 0.03, s_c = 50$, coefficient of $\beta = 0.15$ and manufacturing cost is $c = 950$. The model's optimum outputs are shown in the following tables.

Table 1: Individual Pricing Strategy

Optimal	R_{11}	R_{12}	R_{13}	R_{14}	D_1	D_2	M
Price	1507.27	1506.16	1508.31	1504.01	1357.12	1356.6	1352.35
Time	425	420	430	428	-	-	-
EOQ	66721.22	64231.19	61889.01	65651.52	-	-	-
Profit	23471.22	23203.6	22143.59	22508.25	616724.9	580804.4	2160082.95

Table 2: Collective Pricing Strategies

Optimal	R_{11}	R_{12}	R_{13}	R_{14}	D_1	D_2	M
Price	1277.08	1268.62	1271.08	1266.78	-	-	-
Time	455	451	445	449	-	-	-
EOQ	180645	177978	185113	181077	-	-	-
Profit	-	-	-	-	-	-	507616

5.1 Sensitivity Analysis

Corollary 5.1 i_j^{th} retailer's profit function is strictly increasing function for the coefficient β i.e., $\frac{\partial z_{ij}^R}{\partial \beta} > 0$, if $\left((s_{ij}^R - w_j^D) - \frac{h\delta T}{2} < 0 \right)$ and $(s^m - s_{ij}^R) < 0$.

i_j^{th} retailer's profit function shows positive behaviour with respect to the increment of the coefficient β , if $\delta, h, T, s_i^R, w_j^D$ must follow the identity $\frac{h\delta T}{2} > (s_m - w_j^D)$, otherwise increment of the coefficient β , impacted negatively on the i_j^{th} retailer's profit function. If δ, h are constants then T can be taken in the interval $T \in \left(\frac{3(s^m - w_j^D)}{\delta h}, \infty \right)$.

Corollary 5.2 i_j^{th} retailer's profit function is strictly decreasing function for the coefficient α , i.e. $\frac{\partial z_{ij}^R}{\partial \alpha} < 0$, if $(s_m - s_{ij}^R) < 0$ and $\left(\frac{h\delta T}{2} - 1 \right) > 0$.

i_j^{th} retailer's profit function shows negative behaviour with respect to the increment of the coefficient α , if $\delta, h, T, s_i^R, w_j^D$ must follow the identity $(s_m - s_{ij}^R) < 0$ and $\left(\frac{h\delta T}{2} - 1 \right) > 0$.

It reveals that the suggested selling price by manufacturer is always less than the i_j^{th} retailer's selling price.

Corollary 5.3 i_j^{th} retailer's profit function is strictly increasing function for the initial demand

a_{ij} i.e. $\frac{\partial z_{ij}^R}{\partial a_{ij}} > 0$, if $(s_{ij}^R - w_j^D) > \frac{h\delta T}{2}$.

Result of corollary 5.3 shows that the i_j^{th} retailer's can be benefited by growing market of an item,

he can maintain the business hour in the interval $T \in \left(0, \frac{2(s_{ij}^R - w_j^D)}{\delta h} \right)$

Corollary 5.4 i_j^{th} retailer's profit function is strictly increasing function for the coefficient b_{ij} , i.e.

$\frac{\partial z_{ij}^R}{\partial a_{ij}} > 0$, if $\frac{h\delta T}{3} > \frac{(s_{ij}^R - w_j^D)}{2}$.

Result of corollary 5.4 reveals that replenishment time T is taken from the interval $T \in$

$\left(\frac{3(s_{ij}^R - w_j^D)}{2}, \infty \right)$ then the i_j^{th} retailer's profit function shows positive behaviour for the

coefficient b_{ij} in linear declining market demand of the items.

As per the above discussion, the following conclusions are drawn

- a. The suggested selling price can be helpful to increase the selling price if it can be kept always less than retailers' selling price and greater than wholesale price.

- b. The reorder frequency could be increased to increase the business, if the suggested selling price would be equal to wholesale price of distributors.
- c. We can protect the profit function from the negative influence of alpha if we follow the identities $(s^m - s_{ij}^R) < 0$ and $\left(\frac{h\delta T}{2} - 1\right) > 0$.
- d. The benefit of the initial demand of the item will be available only if we maintain the replenishment time in the interval $T \in \left(0, \frac{2(s_{ij}^R - w_j^d)}{\delta h}\right)$.

6. Conclusion

We have designed a three-layer coordinated multi-channel echelon supply chain for two different pricing strategies: the first one is individual, and the second one is collective. This study aims to decide which pricing strategy performs better for all supply chain members in a non-competitive environment. We have found an optimal profit for all supply chain members by using selling price and finite replenishment time for retailer's end as decision variable in both individual and collective strategies. We have also optimized initial lot size for retailer's wholesale prices for manufacturers and distributors. The suggestions are given by in forms of propositions and numerical examples. This study's theoretical and practical contribution is how to make coordination strategies among multi-echelon supply chain members time-dependent linear decreasing demand. This study is recommended for the inventory manager of supply chain to make a contract to share total profit among manufacturers, distributors and retailers better outcomes. Study concluded that a individual pricing strategy performing better than collective strategy, therefore it is beneficial for managerial purpose in practice. Based on sensitivity analysis, the manufacturer should maintain the suggested selling price less than actual selling price.

The future research scope of this study can be extended by using competitive environment among echelon members. One can be extended this model by incorporating trade credit financing scheme. Model may be also extended by incorporating marketing efforts at retailers' end. It can also be extended by incorporating promotional cost sharing among manufacturers and retailers or manufacturers and distributors.

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