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Analysis of An Inventory Model for Linear Demand with Two Levels of Production Under Influence Rate

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Abstract

This article presents an analysis of an inventory model in the context of linear demand with dual production levels, focusing on deteriorating items and the impact of inflation. This study investigates the intricate relationship between inventory management and economic factors, offering insights into optimizing production decisions and inventory control strategies. The research develops into a scenario where a business faces two production options: a lower level for regular demand and a higher level to address fluctuations or surges.A significant aspect of this study is the incorporation of inflation's influence on inventory dynamics. Inflation can significantly affect production costs, demand, and the overall value of inventory. The analysis examines the interplay between inflation and the optimal production quantity, aiming to develop strategies that maintain inventory levels while mitigating the impact of inflation on costs and profits. By applying mathematical formulations and optimization techniques, this research offers quantitative insights into the optimal order quantities, replenishment strategies, and pricing policies that can be adopted to address the challenges posed by deteriorating items and inflation. Thestudy's findings provide businesses with a comprehensive understanding of how to strategically manage inventory levels and production decisions to navigate the challenges posed by deteriorating items and inflation, ultimately leading to improved operational efficiency and financial performance.

Keywords: Production, Inventory,Linear Demand, Inflation, Deterioration.

Mathematical Subject Classification (2020): 90B05, 90C30, 90B035

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1.Introduction

An inventory is a stockpile of goods or resources maintained by a business or organization to meet future demands or fulfill operational needs. It serves as a buffer between the production and consumption of goods, helping to bridge gaps in supply and demand. inventory management is a critical component of contemporary business operations. Striking the right balance in inventory levels directly impacts a company's ability to meet customer demand, control costs, and ultimately thrive in a competitive market. By leveraging technology and adopting sound inventory management practices, businesses can optimize their operations and maximize profitability.Effective inventory management is essential for achieving smooth operations and customer satisfaction. Tenget al.Teng et al. [1] address the limitations of their model by incorporating not only the cost associated with lost sales but also accounting for the variability in purchase costs, which are not constant. Singh et al. [2] have developed an Economic Order Quantity (EOQ) inventory model for a deteriorating item. In this model, the demand rate follows a deterministic and two-staged pattern. Specifically, it remains constant during the first part of the cycle and becomes a linear function of time during the second part. Additionally, the deterioration rate is proportional to time. Liao [3] has studying the management of an exponentially deteriorating item, considering a scenario where the retailer both receives trade credit from the supplier and offers trade credit to the customer simultaneously. The objective is to minimize the average total cost in this situation. Shaikh [4] described in their model that the demand function is influenced by both price and stock levels. However, during shortage periods, the demand depends solely on the price of the product. The price of the product, in turn, is determined by various markup rates.Chung and Huang [5] presents a model aimed at determining an optimal ordering policy under circumstances that allow for shortages and delays in payment. It demonstrates that the total annual variable cost function exhibits certain types of convex characteristics. The aim of study of Singh et al. [6] is to implement a production model within the inventory management framework, specifically designed to ascertain demand capacities in the context of multi-item, multi-outlet scenarios.

Deterioration can affect various industries, such as perishable goods, pharmaceuticals, and electronics. It necessitates a different approach to inventory control, as organizations must consider factors like shelf life, obsolescence, and quality degradation. Intention of Thangamand. Uthayakumar [7]is to construct an inventory model tailored to the unique challenges posed by deteriorating items in an environment characterized by inflationary pressures. We will employ a sophisticated discounted cash flow (DCF) methodology, considering a finite planning horizon as

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our analytical foundation. Yang et al. [8] build upon the work of Teng, Chang, Dye, and Hung as presented in their 2002 publication titled "An optimal replenishment policy for deteriorating items with time-varying demand and partial backlogging". Mandal [9] was engaged in the process of formulating the Economic Order Quantity (EOQ) model for an inventory comprising items experiencing degradation at a rate adhering to a Weibull distribution. Problem discussed by Bansal and Ahalawat [10] was characterized by items undergoing deterioration and experiencing demand patterns exhibiting exponential growth coupled with temporal dependencies, all within the context of an inflationary environment.Tripathi et al. [11] endeavors to construct an inventory model in which the vendor extends a credit period to the purchaser contingent upon the purchaser placing substantial orders.Sharma et al. [12] introduces an inventory model tailored to items undergoing deterioration following a Weibull distribution, while considering a demand rate characterized as a function of time with a ramp-like trend. The model permits inventory shortages, which are entirely replenished upon availability. Sharma and Yadav [13] were explored a deterministic inventory model, wherein demand is treated as a deterministic variable following a quadratic rate pattern, while accommodating for the possibility of inventory shortages. Kumar and Rajput [14] in their scholarly work, they have formulated a comprehensive inventory model applicable to deteriorating items featuring a consistent rate of deterioration and a demand pattern exhibiting a ramp-like trend, all while considering consumption rates dependent on the current stock levels. Yadav et al. [15] have introduced an inventory model designed specifically for items experiencing deterioration and managed within a dual warehousing framework. Given the prevailing market conditions, they place significant emphasis on incorporating inflation as a pivotal factor when modeling inventory systems. Sharma and Muhammad [16] have undertaken the calculation of correlation coefficients to establish relationships between parameters and various variables within an inventory model addressing deteriorating items.Singh [17] addressed a production-inventory model where the demand is contingent upon both the stock levels and the selling price of the product. The demand rate shows a linear increase relative to stock and time but declines concerning the selling price of the item. Kumar et al. [18] described an inventory model is produced for immediate transient things with cubical polynomial time function demand rate and pareto type perishable rate with permissible delay in payments.

The concept of Two-layer production in inventory systems refers to a sophisticated approach in managing production and inventory levels. In this strategy, two distinct layers of production are utilized to optimize the supply chain process. The first layer involves the production of a basic inventory to meet steady, predictable demand. The second layer, on the other hand,

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addresses fluctuating or unexpected spikes in demand through flexible production capabilities. This two-layered approach helps organizations maintain efficiency in routine operations while efficiently responding to variations in customer demand, ultimately enhancing their competitiveness and adaptability in dynamic market conditions. Singh et al. [19] forwarded a dynamic flow shop scheduling model tailored to the production of prefabricated components. This model considers the dynamic nature of demand, including factors such as advancing due dates, the sudden insertion of urgent components, and order cancellations. Meena et al. [20]and Sunita et al. [21]have constructed a sophisticated inventory system for non-instantly degradable products. This system is characterized by a demand influenced by price sensitivity and incorporates a Weibull-based approach to credit term allocation reduction rates.

Inflation's impact on inventory systems is a critical consideration for businesses. As inflation raises the general price level, it affects the costs associated with inventory control, such as raw materials and storage expenses. Adapting inventory strategies, such as just-in-time inventory or price hedging, is necessary to mitigate the negative effects of inflation and maintain financial stability in an inflationary economic environment.Singh and Sharma [22] introduced a mathematical inventory model that incorporates demand as a function of the selling price, highlighting its dependency on pricing. Additionally, the model assumed that holding costs follow a linear relationship with time in an inflationary setting.Sharma et al. [23] presents three inventory models in their article with considering constant deterioration and partial backlogging, incorporating different demand functions in each model. The study also explores three types of replenishment strategies: individual replenishment, joint replenishment, and a combination of both individual and joint replenishments.Singh et al. [24] examined a production inventory model with three degrees of complexity, specifically designed for situations involving a constant rate of deterioration. This model plays a crucial role in the production process of both boards and assembly units.

After a thorough review of numerous research articles, we have conducted a comprehensive comparative analysis of various scholars who have employed distinct models to investigate demand patterns, production levels, inflation, and associated variables. The outcomes of this analysis are succinctly presented in Table 1 below:

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Table 1: Summarized Analysis of Some Authers Research Work

Authors	**EOQ/ EPQ Model**	**Demand Pattern**	**Productio n Level**	**Deterioration Rate**	**Inflation Rate**
Singhet al. [2]	EOQ	Quadratic	No	constant	No
Shaikh [4]	EOQ	Price and stock dependent	No	Non instantaneous	Yes
Singh et.al.[6]	EPQ	Constant	Two	Deterministic	No
Mandal [9]	EOQ	Ramp Type	No	Weibull Distribution	No
Bansal and Ahlawat [10]	EOQ	Exponential	No	Constant	Constant
Kumar et al. [18]	EOQ	Cubical Polynomial Time Function	No	Pareto Type	No
Singh and Sharma [22]	EOQ	Price Dependent	No	Constant	Yes
Singh et.al.[24]	EPQ	Quadratic time function	Three	Constant	No

This table provides a condensed overview of a comparative analysis conducted by several researchers, each employing distinct model types to explore aspects such as demand patterns, production levels, inflation, and related variables. The research findings reveal a wide range of insights, with each model type presenting distinct advantages in enhancing our comprehension of the intricate dynamics underlying these economic phenomena.We will now delve into an essential Production Inventory Model, which considers various parameters discussed in Table 1, including a linear demand function, a two-level production strategy, and the management of deteriorating items within an inflationary context.

In this paper we use following 7 sections:-

Section 2-Notations and Assumptions of our Inventory Model displayed in this Section.

Section 3-Mathematical Development and Solution of our Model are in this Section.

Section 4-Solution algorithm and Numerical Example with taken special values of parameters used in this Inventory Model.

Section 5-Sensetivity of our problem shown by a table, which was made by changes values of given parameters used in this Model.

Section 6- Graphs of observations of model given in this Section.

Section 7-Graphical Conclusion is given in this Section.

Section 8-Conlusion of our Model is cleared in this Section.

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2.Assumption and Notations

2.1 Assumptions

(i) Demand rate is influenced by time and the linear time function.

(ii) The deterioration rate is taken as constantfunction with respect to time.

(iii) Lead time is assumed to be zero.

(iv) Holding cost is constant.

(v) The rate of inflation varies throughout time.

(vi) The replenishment rate is assumed to be limitless and immediate.

(vii) Two-layer production considered.

2.2 Notations

(i) C_P: Production cost per unit.

(ii) C_S: Ordering (Set up) Cost per unit.

(iii) C_H: Unit holding cost per unit time.

(iv) C_D: Deterioration Cost per unit.

(v) C_B: Purchase cost per unit.

(vi) ω: Deterioration Rate, where $0 < \omega < 1$

(vii) T_3: Cycle length.

(viii) D(t): The demand rate, $D(t) = a + bt$, where a, b> 0

(ix) $I_1(t)$: Inventory level at production time during interval $[0, T_1]$.

(x) $I_2(t)$: Inventory level during time interval $[T_1, T_2]$.

(xi) $I_3(t)$: Inventory level during shortages in time interval $[T_2, T_3]$.

(xii) Q*: Order quantity during the cycle length T_3.

(xiii) I_{M1}: Inventory level at time T_1.

(xiv) I_{M2}: Maximum Inventory level during $[0, T_3]$.

(xv) r(t): Inflationrate, $r(t) = e^{\theta t}$, Where θis inflation rate parameter.

(xvi) P:Rate of production per unit time.

(xvii) $C_T(T_3)$: Total Cost during complete cycle time.

(xviii) w.r.t.: With respect to

We use above assumptions and notations,then we construct a mathematical inventorymodel in which we try to find an optimal solution for EPQ.

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3 Mathematical Formulations of The Proposed Model

As shown in Figure-1, The manufacturing process begins at $t=0$ andcontinues until the stock amount reaches its peak at $t=T_1$. During this time interval, the inventory levelis denoted by $I_1(t)$. In the time interval $t=T_1$ to $t=T_2$ rate of production change and inventory level in this interval denoted by $I_2(t)$. And next in time interval $t=T_2$ to $t=T_3$ Inventory level gradually decreases and reaches zero at time $t=T_3$ due to customer demand and impact of deterioration on items. The inventory level during time interval $[T_2,T_3]$ is denoted by $I_3(t)$. Inventory level at time $t=0$ the Inventory level is zero, at time $t=T_1$ the Inventory level is I_{M1} and at time $t=T_2$ Inventory level is I_{M2} At time $t=T_3$ the Inventory level is zero.During the time interlude, $[0,T_1]$ the production rate is taken as and demand rates is $D(t)=a+bt$ where a andb are positive value and D is less than P. During time interval $[T_1,T_2]$ mounting rate to be measured as $\kappa(P-D)$ where κ is constant and $(\kappa>1)$.

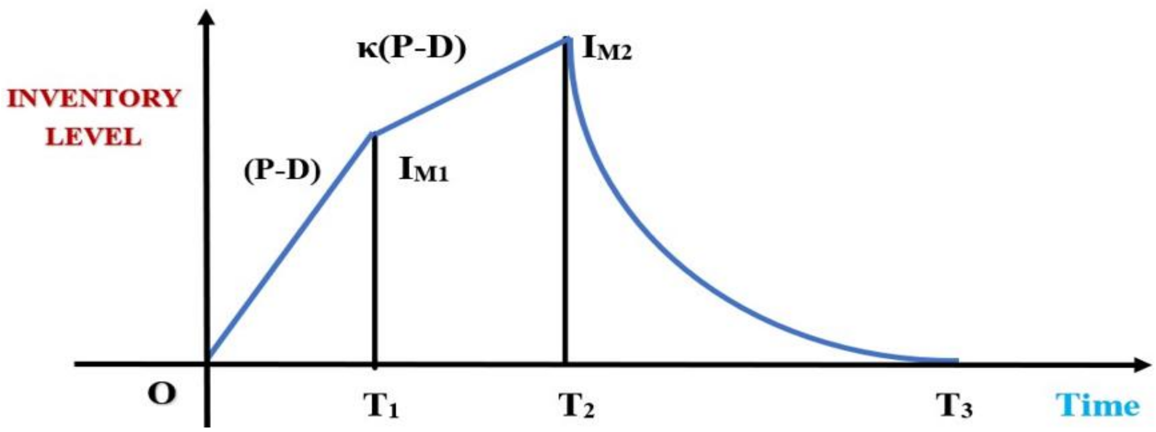

Figure 1: Two level Production Inventory Model for Proposed Model

3.1. Mathematical Formulation of The Model

The differential equation during the time interval $[0,T_1]$ is presented as follows:

$$\frac{dI(t)}{dt}+\omega I(t)=P-(a+bt); \qquad 0\le t\le T_1 \qquad (1)$$

The differential equation during the time interval $[T_1,T_2]$is presented as follows: -

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$$\frac{dI(t)}{dt}+\omega I(t)=\kappa\{P-(a+bt)\}; \qquad T_1 \le t \le T_2 \quad (2)$$

The differential equation during the time interval $[T_2, T_3]$is presented as follows: -

$$\frac{dI(t)}{dt}+\omega I(t)=-(a+bt); \qquad T_2 \le t \le T_3 \quad (3)$$

With Boundary Conditions

$$I(0)=0,\ I(T_1)=I_{M1},\ I(T_2)=I_{M2},\ I(T_3)=0 \quad (4)$$

Solution of equation (1) with help of boundary conditions given in (2), we get

$$I(t)=\frac{1}{\omega^2}\left[(P\omega-a\omega+b)\left(1-e^{-\omega t}\right)-bt\omega\right] \qquad 0 \le t \le T_1 \quad (5)$$

$$I(t)=\frac{\kappa}{\omega^2}\left[(P\omega-a\omega+b)\left(1-e^{-\omega t}\right)-bt\omega\right] \qquad T_1 \le t \le T_2 \quad (6)$$

$$I(t)=\frac{1}{\omega^2}\left[-a\omega-b(\omega t-1)+e^{\omega(T_3-t)}\left\{a\omega+b(\omega T_3-1)\right\}\right] \qquad T_2 \le t \le T_3 \quad (7)$$

Maximum inventory level R_1: The maximum inventory level during the period (0, T_1) is solved as followsfrom the Eqs. (4)–(5), $I(T_1) = I_{M1}$

$$I_{M1}=\frac{1}{\omega^2}\left[(P\omega-a\omega+b)\left(1-e^{-\omega T_1}\right)-bT_1\omega\right] \qquad (8)$$

Maximum inventory level R_2: The maximum inventory level during the period (0, T_1) is solved as follows

from the Eqs. (4)–(6), $I(T_2) = I_{M2}$

$$I_{M2}=\frac{\kappa}{\omega^2}\left[(P\omega-a\omega+b)\left(1-e^{-\omega T_2}\right)-bT_2\omega\right] \qquad (9)$$

3.2Cost Calculation of Proposed Model

To calculate cost of the proposed model we take different types of costs, which are follows:

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(3.2.1)The Holding Cost (H.C.) $= \int_0^{T_3} C_H.\{I(t)\}.e^{-\theta t}dt$

$$= \int_0^{T_1} C_H.\{I(t)\}.e^{-\theta t}dt + \int_{T_1}^{T_2} C_H.\{I(t)\}.e^{-\theta t}dt + \int_{T_2}^{T_3} C_H.\{I(t)\}.e^{-\theta t}dt$$

$$H.C. = \left[\begin{array}{l} \frac{C_H(P\omega - a\omega + b)}{\omega^2}\left\{\frac{e^{-(\omega+\theta)T_1}}{(\omega+\theta)} - \frac{e^{-\theta T_1}}{\theta} + \frac{1}{\theta} - \frac{1}{(\omega+\theta)}\right\} + \frac{C_H b}{\omega}\left\{\frac{e^{-\theta T_1}}{\theta^2}(T_1+1) - \frac{1}{\theta^2}\right\} \\ + \frac{C_H \kappa(P\omega - a\omega + b)}{\omega^2}\left\{\frac{e^{-(\omega+\theta)T_2} - e^{-(\omega+\theta)T_1}}{(\omega+\theta)} - \frac{e^{-\theta T_2} - e^{-\theta T_1}}{\theta}\right\} + \frac{C_H \kappa b}{\omega}\left\{\frac{e^{-\theta T_2}}{\theta^2}(T_2+1)\right. \\ \left. - \frac{e^{-\theta T_1}}{\theta^2}(T_1+1)\right\} + \frac{C_H(b - a\omega)}{\omega^2\theta}\left(e^{-\theta T_2} - e^{-\theta T_3}\right) + \frac{C_H b}{\omega\theta^2}\left\{e^{-\theta T_3}(T_3+1) - e^{-\theta T_2}(T_2+1)\right\} \\ + \frac{C_H\{a\omega + b(\omega T_3 - 1)\}e^{\omega T_3}}{\omega^2(\omega+\theta)}\left(e^{-(\omega+\theta)T_2} - e^{-(\omega+\theta)T_3}\right) \end{array}\right] \quad (10)$$

(3.3.2)The deterioration cost(D.C.) $= C_D.\omega\int_0^{T_3} I(t).e^{-\theta t}dt$

$$D.C. = \left[\begin{array}{l} \frac{C_D(P\omega - a\omega + b)}{\omega}\left\{\frac{e^{-(\omega+\theta)T_1}}{(\omega+\theta)} - \frac{e^{-\theta T_1}}{\theta} + \frac{1}{\theta} - \frac{1}{(\omega+\theta)}\right\} + C_D b\left\{\frac{e^{-\theta T_1}}{\theta^2}(T_1+1) - \frac{1}{\theta^2}\right\} \\ + \frac{C_D\omega\kappa(P\omega - a\omega + b)}{\omega}\left\{\frac{e^{-(\omega+\theta)T_2} - e^{-(\omega+\theta)T_1}}{(\omega+\theta)} - \frac{e^{-\theta T_2} - e^{-\theta T_1}}{\theta}\right\} + C_D \kappa b\left\{\frac{e^{-\theta T_2}}{\theta^2}(T_2+1)\right. \\ \left. - \frac{e^{-\theta T_1}}{\theta^2}(T_1+1)\right\} + \frac{C_D\omega\kappa(P\omega - a\omega + b)}{\omega}\left\{\frac{e^{-(\omega+\theta)T_2} - e^{-(\omega+\theta)T_1}}{(\omega+\theta)} - \frac{e^{-\theta T_2} - e^{-\theta T_1}}{\theta}\right\} + \\ C_D \kappa b\left\{\frac{e^{-\theta T_2}}{\theta^2}(T_2+1) - \frac{e^{-\theta T_1}}{\theta^2}(T_1+1)\right\} + \frac{C_D(b - a\omega)}{\omega\theta}\left(e^{-\theta T_2} - e^{-\theta T_3}\right) + \frac{C_D b}{\theta^2}\left\{e^{-\theta T_3}\right. \\ \left.(T_3+1) - e^{-\theta T_2}(T_2+1)\right\} + \frac{C_D\{a\omega + b(\omega T_3 - 1)\}e^{\omega T_3}}{\omega(\omega+\theta)}\left(e^{-(\omega+\theta)T_2} - e^{-(\omega+\theta)T_3}\right) \end{array}\right] \quad (11)$$

(3.3.3)Purchase Cost (P_r.C.)= C_B.Q*

$$C_B.Q^* = \frac{C_B\kappa}{\omega^2}\left[(P\omega - a\omega + b)\left(1 - e^{-\omega T_2}\right) - bT_2\omega\right] \quad (12)$$

(3.2.4)Production Cost (P.C.) $= C_P.(a + bt)$ (13)

(3.2.5)Ordering Cost (O.C.) = C_S (14)

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(3.3.6) Total Cost (T.C.) = $C_T(T_3) = [\text{P.C.} + O.C. + H.C. + D.C. + P_r.C.] =$

$$T.C. = \left[\begin{array}{l} C_P.(a+bt) + C_S + \frac{(C_H+\omega C_D)(P\omega - a\omega + b)}{\omega^2}\left\{\frac{e^{-(\omega+\theta)T_1}}{(\omega+\theta)} - \frac{e^{-\theta T_1}}{\theta} + \frac{1}{\theta} - \frac{1}{(\omega+\theta)}\right\} + \frac{(C_H+\omega C_D)b}{\omega} \\ \left\{\frac{e^{-\theta T_1}}{\theta^2}(T_1+1) - \frac{1}{\theta^2}\right\} + \frac{(C_H+\omega C_D)\kappa(P\omega - a\omega + b)}{\omega^2}\left\{\frac{e^{-(\omega+\theta)T_2} - e^{-(\omega+\theta)T_1}}{(\omega+\theta)} - \frac{e^{-\theta T_2} - e^{-\theta T_1}}{\theta}\right\} + \\ \frac{(C_H+\omega C_D)\kappa b}{\omega}\left\{\frac{e^{-\theta T_2}}{\theta^2}(T_2+1) - \frac{e^{-\theta T_1}}{\theta^2}(T_1+1)\right\} + \frac{(C_H+\omega C_D)(b-a\omega)}{\omega^2\theta}\left(e^{-\theta T_2} - e^{-\theta T_3}\right) \\ + \frac{(C_H+\omega C_D)b}{\omega\theta^2}\left\{e^{-\theta T_3}(T_3+1) - e^{-\theta T_2}(T_2+1)\right\} + \frac{(C_H+\omega C_D)\{a\omega + b(\omega T_3 - 1)\}e^{\omega T_3}}{\omega^2(\omega+\theta)}\left\{e^{-(\omega+\theta)T_2}\right. \\ \left. -e^{-(\omega+\theta)T_3}\right\} + \frac{C_B\kappa}{\omega^2}\left\{(P\omega - a\omega + b)\left(1 - e^{-\omega T_2}\right) - bT_2\omega\right\} \end{array}\right] \quad (15)$$

We take a relationship such that $T_1 = \alpha T_3$; $T_2 = \beta T_3$ *and* $t = \gamma T_3$ therefore the total cost will be find in new form in terms of T_3.

$$TC = \left[\begin{array}{l} C_P.(a+b\gamma T_3) + C_S + \frac{(C_H+\omega C_D)(P\omega - a\omega + b)}{\omega^2}\left\{\frac{e^{-(\omega+\theta)\alpha T_3}}{(\omega+\theta)} - \frac{e^{-\alpha\theta T_3}}{\theta} + \frac{1}{\theta} - \frac{1}{(\omega+\theta)}\right\} + \frac{(C_H+\omega C_D)b}{\omega} \\ \left\{\frac{e^{-\theta\alpha T_3}}{\theta^2}(\alpha T_3+1) - \frac{1}{\theta^2}\right\} + \frac{\kappa(C_H+\omega C_D)(P\omega - a\omega + b)}{\omega^2}\left\{\frac{e^{-(\omega+\theta)\beta T_3} - e^{-(\omega+\theta)\alpha T_3}}{(\omega+\theta)} - \frac{e^{-\theta\beta T_3} - e^{-\theta\alpha T_3}}{\theta}\right\} + \\ \frac{(C_H+\omega C_D)\kappa b}{\omega}\left\{\frac{e^{-\theta\beta T_3}}{\theta^2}(\beta T_3+1) - \frac{e^{-\theta\alpha T_3}}{\theta^2}(\alpha T_3+1)\right\} + \frac{(C_H+\omega C_D)(b-a\omega)}{\omega^2\theta}\left(e^{-\theta\beta T_3} - e^{-\theta T_3}\right) + \\ \frac{(C_H+\omega C_D)b}{\omega\theta^2}\left\{e^{-\theta T_3}(T_3+1) - e^{-\theta\beta T_3}(\beta T_3+1)\right\} + \frac{(C_H+\omega C_D)}{\omega^2(\omega+\theta)}\{a\omega + b(\omega T_3 - 1)\}\left(e^{(\omega-\omega\beta-\theta\beta)T_3}\right. \\ \left. -e^{-\theta T_3}\right) + \frac{C_B\kappa}{\omega^2}\left[(P\omega - a\omega + b)\left(1 - e^{-\omega\beta T_3}\right) - b\beta T_3\omega\right] \end{array}\right] \quad (16)$$

Objective of The Proposed Model

The study's objective is to investigate the optimal time T_3, which minimizes TC as follows and then differentiates with respect to T_3.We get $\frac{d}{dT_3}(T.C.) =$

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$$\frac{d}{dT_3}(T.C.)=\left[\begin{array}{l}\frac{(C_H+\omega C_D)(P\omega-a\omega+b)}{\omega^2}\left\{-\alpha e^{-(\omega+\theta)\alpha T_3}+\alpha e^{-\alpha\theta T_3}\right\}+\frac{(C_H+\omega C_D)b}{\omega}\left\{\frac{-\alpha e^{-\theta\alpha T_3}}{\theta}\right.\\ \left.(\alpha T_3+1)+\frac{\alpha e^{-\theta\alpha T_3}}{\theta^2}\right\}+\frac{\kappa(C_H+\omega C_D)(P\omega-a\omega+b)}{\omega^2}\left\{-\beta e^{-(\omega+\theta)\beta T_3}+\alpha e^{-(\omega+\theta)\alpha T_3}\right.\\ \left.+\beta e^{-\theta\beta T_3}-\alpha e^{-\theta\alpha T_3}\right\}+\frac{(C_H+\omega C_D)\kappa b}{\omega}\left\{\frac{-\beta e^{-\theta\beta T_3}}{\theta}(\beta T_3+1)+\frac{\beta e^{-\theta\beta T_3}}{\theta^2}+\frac{\alpha e^{-\theta\alpha T_3}}{\theta}\right.\\ \left.(\alpha T_3+1)-\frac{\alpha e^{-\theta\alpha T_3}}{\theta^2}\right\}+\frac{(C_H+\omega C_D)(b-a\omega)}{\omega^2\theta}\left(-\theta\beta e^{-\theta\beta T_3}+\theta e^{-\theta T_3}\right)+\frac{(C_H+\omega C_D)}{\omega^2(\omega+\theta)}\\ \left(b\omega\left\{e^{(\omega-\omega\beta-\beta\theta)T_3}+e^{-\theta T_3}\right\}+(a\omega-b+\omega T_3)\left\{(\omega-\omega\beta-\beta\theta)e^{(\omega-\omega\beta-\beta\theta)T_3}+\theta e^{-\theta T_3}\right\}\right)\\ +\frac{(C_H+\omega C_D)b}{\omega\theta^2}\left\{-\theta e^{-\theta T_3}(T_3+1)+e^{-\theta T_3}+\theta\beta e^{-\theta\beta T_3}(\beta T_3+1)-\beta e^{-\theta\beta T_3}\right\}+C_Pb\gamma\\ +\frac{C_B\kappa}{\omega^2}\left\{(P\omega-a\omega+b)\left(\omega\beta e^{-\omega\beta T_3}\right)-b\beta\omega\right\}+C_Pb\gamma\end{array}\right] \quad (17)$$

Again, differentiates total cost with respect to T_3. We get $\frac{d^2}{dT_3^2}(T.C.)=$

$$\frac{d^2}{dT_3^2}(T.C.)=\left[\begin{array}{l}\frac{(C_H+\omega C_D)(P\omega-a\omega+b)}{\omega^2}\left\{\alpha^2e^{-(\omega+\theta)\alpha T_3}-\alpha\theta^2e^{-\alpha\theta T_3}\right\}+\frac{(C_H+\omega C_D)b}{\omega}\\ \left\{\alpha^2e^{-\theta\alpha T_3}(\alpha T_3+1)-\frac{2\alpha^2e^{-\theta\alpha T_3}}{\theta}\right\}+\frac{\kappa(C_H+\omega C_D)(P\omega-a\omega+b)}{\omega^2}\\ \left\{\beta^2(\omega+\theta)e^{-(\omega+\theta)\beta T_3}-\alpha^2(\omega+\theta)e^{-(\omega+\theta)\alpha T_3}-\theta\beta^2e^{-\theta\beta T_3}+\alpha^2\theta e^{-\theta\alpha T_3}\right\}\\ +\frac{(C_H+\omega C_D)\kappa b}{\omega}\left\{\frac{2\alpha^2e^{-\theta\alpha T_3}}{\theta}+\beta^2e^{-\theta\beta T_3}(\beta T_3+1)-\alpha^2e^{-\theta\alpha T_3}(\alpha T_3+1)\right.\\ \left.-\frac{2\beta^2e^{-\theta\beta T_3}}{\theta}\right\}+\frac{(C_H+\omega C_D)(b-a\omega)}{\omega^2\theta}\left(\theta^2\beta^2e^{-\theta\beta T_3}-\theta^2e^{-\theta T_3}\right)+\frac{(C_H+\omega C_D)}{\omega^2(\omega+\theta)}\\ \left[(b\omega+\omega)\left\{(\omega-\omega\beta-\beta\theta)e^{(\omega-\omega\beta-\beta\theta)T_3}-\theta e^{-\theta T_3}\right\}+(a\omega-b+\omega T_3)\right.\\ \left.\left\{(\omega-\omega\beta-\beta\theta)^2e^{(\omega-\omega\beta-\beta\theta)T_3}-\theta^2e^{-\theta T_3}\right\}\right]+\frac{(C_H+\omega C_D)b}{\omega\theta^2}\left\{\theta^2e^{-\theta T_3}(T_3+1)\right.\\ \left.-2\theta e^{-\theta T_3}-\theta^2\beta^2e^{-\theta\beta T_3}(\beta T_3+1)+2\theta\beta^2e^{-\theta\beta T_3}\right\}-\frac{C_B\kappa}{\omega^2}(P\omega-a\omega+b)\\ \left(\omega^2\beta^2e^{-\omega\beta T_3}\right)\end{array}\right]$$

(18)

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By using Mathematica -Software -9 we find that $\frac{d^2}{dT_3^2}(T.C.) > 0$ for the optimal value of T_3=3.2343.

This Value of T_3 minimizes Total Cost of our Inventory Model.

4.1 Solution Algorithm: -

The solution algorithm of our proposed model is given below.

Step 1. Calculate different type of Costs using Equation (10) to (15).

Step 2. Calculate Total average Inventory Cost using equation (16).

Step 3. Find first derivative of T.C. w.r.t. T_3. And after find first derivative calculate positive value of T_3. We find the value of T_3 is 3.2343.

Step 4. Calculate $\frac{d^2(T.A.C.)}{dT_3^2}$ and we find that $\frac{d^2(T.A.C.)}{dT_3^2} > 0$ at the point T_3=3.2343

Step 5. Prepared the mathematical software MATLAB (R2021a) and putting the value of a=1.5, b=2.5, α=0.45, β=0.65, γ=1.5, ω=0.7, θ=0.20, C_H= 120, C_D= 130, C_B=110, C_P =120, C_S=115, in equation of TAC.

Step 6. After solving step 5 we find the value of TAC is 193363

Step 7. Change values of different parameters with rate of +20%, +10%, -10%, -20% and find different results. From these results we construct sensitivity analysis.

Step 8. After that we plots different graphs of TAC with different parameters to validate the proposed model.

Step 9. Result

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Numerical Example

4.2 Numerical Example Number 1

Should we opt to employ a numerical illustration and avail ourselves of certain parameter values intrinsic to our inventory model, namely: -

a=1.5, b=2.5, α=0.45, β=0.65, γ=1.5, ω=0.7,θ=0.20, C_H= 120, C_D= 130,C_B=110, C_P =120, C_S=115,

Subsequently, we apply these specified values within the confines of Equation (16) and Equation(18). We employ the computational prowess of the mathematical software MATLAB (versionR2021a) to resolve this matter. The outcomes thereby obtained reveal the following optimal values:

P	T_1	T_2	T_3	T	I_{M1}	I_{M2}	TC
100	1.4554	2.10229	3.2343	4.85145	219.036	1824.84	193363

From above example we tried to show the mathematical approach of the given system of inventory model.

4.3 Numerical Example Number 2

Should we opt to employ a numerical illustration and avail ourselves of certain parameter values intrinsic to our inventory model, namely:

a=1.5, b=2.5, α=0.45, β=0.65, γ=1.5, ω=0.7, θ=0.20, C_H= 120, C_D= 130, C_B=110, C_P =120, C_S=115,

Subsequently, we apply these specified values within the confines of Equation (16) and Equation (18). We employ the computational prowess of the mathematical software MATLAB (version R2021a) to resolve this matter. The outcomes thereby obtained reveal the following optimal values:

P	T_1	T_2	T_3	T	I_{M1}	I_{M2}	TC
200	1.3922	2.01105	3.09394	4.6408	413.461	3426.17	347697

From above example we tried to show the mathematical approach of the given system of inventory model.

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5 Sensitivity Analysis

For sensitivity analysis of this Model, we change values of parameters one by one and announce the effects **T, I_{M1}, I_{M1}, TC** Rate of changes (in percentage) in values of parameters are taken -20 %, -10%, +10% and +20%. Following table show result of above changes.

Parameter		**T_1**	**T_2**	**T_3**	**T**	**I_{M1}**	**I_{M2}**	**TC**
P=100	+20 %	1.435478	2.073468	3.18995	4.784925	258.143	2146.96	224411
	+10%	1.444689	2.086773	3.21042	4.81563	238.616	1986.12	208908
	-10%	1.47006	2.12342	3.2668	4.9002	199.411	1663.18	177777
	-20%	1.483767	2.143219	3.29726	4.94589	179.716	1500.94	162135
ω=0.7	+20 %	1.324179	1.912703	2.94262	4.41393	205.164	1721.62	167198
	+10%	1.385793	2.001701	3.07954	4.61931	211.728	1770.62	179264
	-10%	1.534959	2.217163	3.41102	5.11653	227.242	1885.3	210080
	-20%	1.626471	2.349347	3.61438	5.42157	236.475	1952.68	230201
θ=0.2	+20 %	1.32003	1.90671	2.9334	4.4001	192.273	1588.45	162728
	+10%	1.382112	1.996384	3.07136	4.60704	204.376	1695.09	176097
	-10%	1.544265	2.230605	3.4317	5.14755	237.305	1987.34	216558
	-20%	1.654875	2.390375	3.6775	5.51625	260.842	2197.9	249236
C_H=120	+20 %	1.4337	2.0709	3.1860	4.779	216.717	1804.28	209993
	+10%	1.44396	2.08572	3.2088	4.8132	216.717	1804.28	200764
	-10%	1.468485	2.121145	3.2633	4.89495	221.68	1848.34	185941
	-20%	1.48338	2.14266	3.2964	4.9446	224.723	1875.33	178484
C_D=130	+20 %	1.47591	2.13187	3.2798	4.9197	223.19	1861.77	182091
	+10%	1.4652	2.1164	3.256	4.884	221.01	1842.41	187741
	-10%	1.446615	2.089555	3.2147	4.82205	223.19	1861.77	203915
	-20%	1.438515	2.077855	3.1967	4.79505	215.62	1794.55	204575
C_P=120	+20 %	1.456335	2.103595	3.2363	4.85445	219.21	1826.46	193882
	+10%	1.455885	2.102945	3.2353	4.85295	219.12	1825.65	193597
	-10%	1.454985	2.101645	3.2333	4.84995	218.945	1824.03	193126
	-20%	1.454535	2.100995	3.2323	4.84845	218.854	1823.22	192891
κ=5	+20 %	1.426905	2.061085	3.1709	4.75635	213.288	2128.68	222553
	+10%	1.4400	2.0800	3.200	4.800	215.91	1976.92	207939
	-10%	1.47375	2.12875	3.2750	4.9125	222.75	1672.08	178810
	-20%	1.49589	2.16073	3.3242	4.9863	227.28	1518.49	164303

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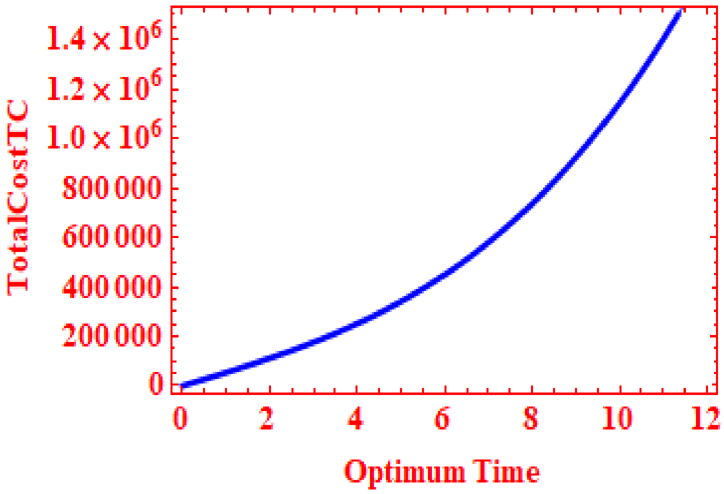
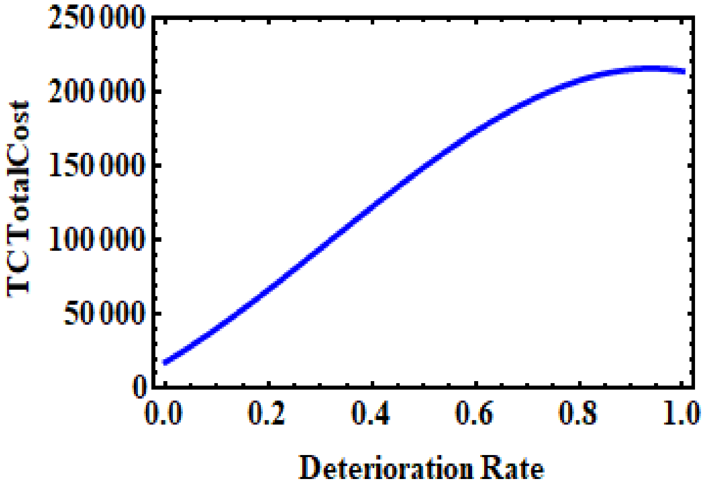
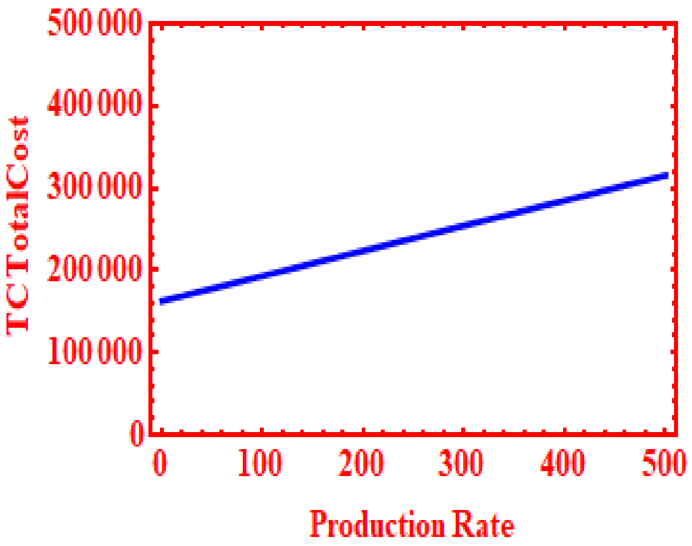
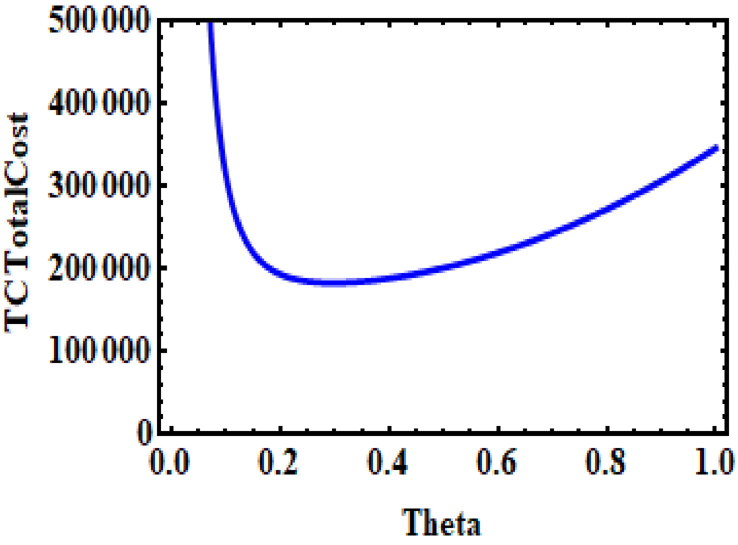
6. Graphs of ObservationsandResults

To validate the proposed model, we have plot graphs w.r.t. to Total Cost and Optimal Time.









7. Graphical Conclusion

Graphical illustration serves as a means of dissecting mathematical data, effectively unveiling the interplay among data points, concepts, and notions within a visual representation. It arguably stands as one of the foremost pedagogical approaches, its effectiveness often contingent upon the volume of information within a given domain. Upon a thorough examination of the graphs and tables pertaining to our Production Inventory Model, a multitude of diverse outcomes come to the fore. In this discourse, we shall delve into several pivotal conclusions of significance.

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(i) We observe that graph Optimum Total Cost w.r.t.Optimal Time Cycle isslightly increasing with convexity.

(ii) Graphs of Optimum total Cost is increasing in starting some time with respect to Deterioration rate parameter andafter some time it is decreasing. The graph is concave type.

(iii) Graph of Optimum total Cost is increasing with Production rate parameter and graph is straight line.

(iv) It is seen that graph of Optimal Total Cost is covex type with respect to Inflation rate parameter.

8 Conclusion

The research conducted on the inventory model for linear demand with two production levels involving deteriorating items in the context of inflation has provided valuable insights and findings. The study explored the complex interplay of factors such as inflation rates and item deterioration within the inventory management framework. This articleincludes a deeper understanding of how inflation impacts inventory decisions, especially in scenarios involving perishable goods. The analysis has shown that inflation can significantly affect optimal ordering policies and overall inventory costs. Furthermore, the study has highlighted the importance of considering multiple production levels when designing inventory models, as this can lead to more effective and cost-efficient inventory management strategies. Overall, this research contributes to the body of knowledge in inventory management by shedding light on the challenges posed by inflation and deteriorating items and by providing practical insights that can inform decision-making in supply chain and inventory management contexts.

Further research in this area may explore additional complexities and refine models to accommodate real-world variations and uncertainties.

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