
Exploring Complex Networks through Multidimensional Analysis

Subham Kumar Mishra

Monu Kumar

Research Scholar, Dept of Mathematics, MMH College Ghaziabad

Parul Tomar

Abstract

Complex networks play a pivotal role in understanding and modeling various interconnected systems, ranging from social networks to biological networks and technological infrastructures. This abstract introduces the concept of exploring complex networks through multidimensional analysis, a multifaceted approach that delves into the intricate structures and dynamics of these networks. Multidimensional analysis extends beyond traditional network analysis techniques by considering not only the topological properties of nodes and edges but also incorporating additional dimensions, such as node attributes, temporal dynamics, and community structures. By integrating these diverse dimensions, researchers can uncover hidden patterns, predict network behavior, and gain a deeper understanding of complex systems. This research provides an overview of the key aspects of multidimensional analysis, including the methods and tools employed for studying complex networks. It highlights the significance of this approach in addressing real-world challenges, such as identifying influential nodes in social networks, predicting disease spread in epidemiological networks, and optimizing communication networks for efficiency and resilience.

Keywords:-Complex Networks, Multidimensional Analysis, Network Analysis, Multilayer Networks, Multidimensional Networks

Introduction

Complex networks serve as a fundamental framework for modeling and understanding a wide array of systems in diverse domains, ranging from social interactions and biological processes to transportation networks and the World Wide Web. The intricacies of these networks are characterized by nodes (representing entities) and edges (representing connections) that collectively shape the structure and dynamics of the system. Traditionally, network analysis has focused on uncovering topological patterns within these networks, such as centrality measures or community detection, to gain insights into their underlying properties. However, in many real-world applications, the complexity of the data transcends mere topological aspects, necessitating a more comprehensive approach.

Multidimensional analysis represents a paradigm shift in the study of complex networks. This multifaceted methodology transcends the confines of traditional network analysis by incorporating multiple dimensions into the analysis process. While topology remains a crucial dimension, multidimensional analysis extends its purview to include various other dimensions, such as node attributes, temporal dynamics, and community structures. This approach acknowledges that real-world networks are not solely defined by their connectivity but are influenced by additional characteristics and evolving patterns.

At its core, multidimensional analysis seeks to answer complex questions that topological analysis alone cannot address. It allows researchers and practitioners to explore networks in a holistic manner, considering the interplay between network structure, node attributes, and temporal evolution. This comprehensive perspective empowers us to unveil hidden patterns, predict network behavior, and extract actionable insights from complex systems. We will delve into the various dimensions that contribute to the complexity of networks, the methods and tools employed in multidimensional analysis, and the real-world implications of this approach. By exploring complex networks through multidimensional analysis, we embark on a journey to unravel the intricate tapestry of interconnected systems, fostering innovation and progress in fields as diverse as sociology, biology, computer science, and engineering. This multidisciplinary perspective serves as a powerful framework for navigating the ever-expanding landscape of network science and big data.

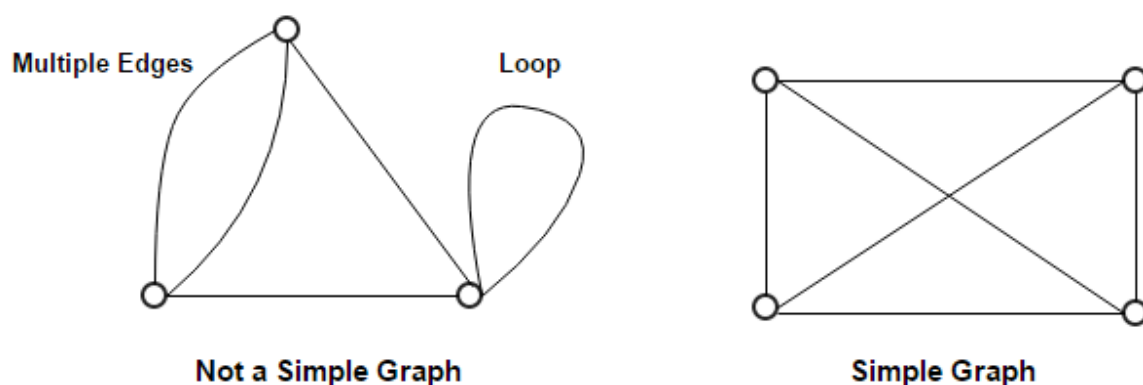


Fig 1 Simple graph and non simple graph with multiple edge

A simple graph is a mathematical representation of a set of vertices (nodes) connected by edges, where each edge connects two distinct vertices, and there are no loops (edges connecting a vertex to itself) or multiple edges (more than one edge between the same pair of vertices). In a simple graph, each edge is unique, and no additional information is associated with edges. In contrast, a non-simple graph allows for multiple edges between the same pair of vertices. This means that you can

have multiple connections or relationships between the same two nodes. These multiple edges could represent different types of relationships or interactions, and they are often used in network analysis to capture more complex connections in various applications, such as social networks or transportation systems.

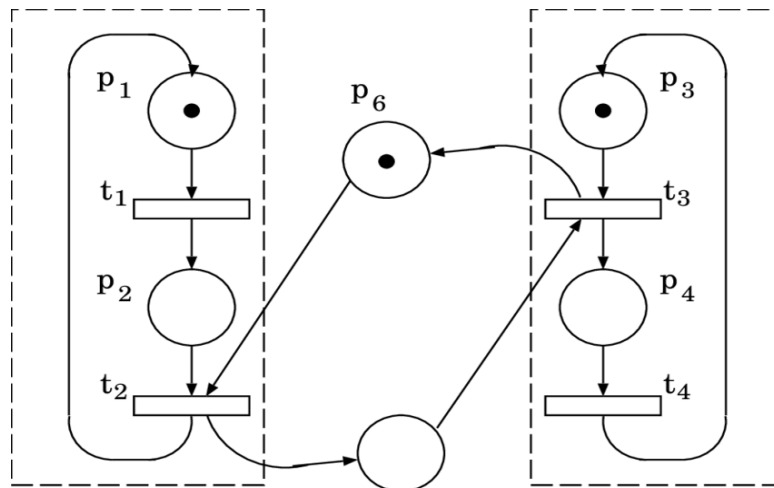


Fig 2 The producer/consumer problem

The producer-consumer problem is a classical synchronization challenge in computer science and operating systems. It involves two types of processes, producers and consumers, that interact through a shared buffer. Producers generate data or items and place them in the buffer, while consumers remove items from the buffer and process them. The key challenge is to ensure proper coordination and synchronization between producers and consumers to prevent issues like buffer overflows or underflows. This problem illustrates the complexities of managing shared resources in a concurrent environment and is addressed using synchronization mechanisms like semaphores, mutexes, and condition variables. Solutions to the producer-consumer problem play a crucial role in designing efficient and reliable multi-threaded or multi-process systems for various applications, from data processing pipelines to task scheduling.

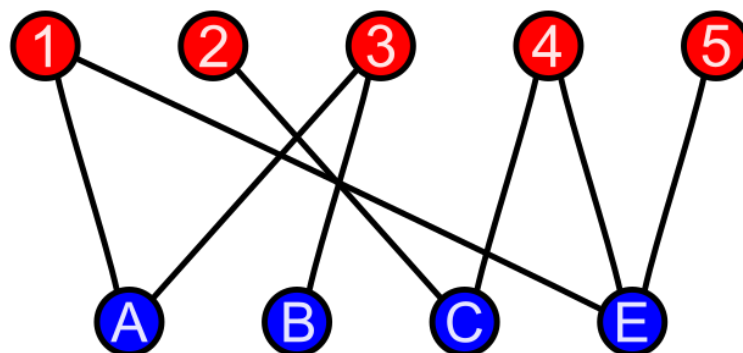


Fig 3 Bipartite Graph

A bipartite graph is a fundamental concept in graph theory, representing a type of graph where vertices can be divided into two distinct sets, often referred to as "partite sets," with edges only connecting vertices from one set to the other. This unique property makes bipartite graphs useful for modeling relationships between two different types of entities or objects. For example, in a bipartite graph modeling a social network, one set of vertices might represent users, and the other set could represent events. Edges between users and events would signify attendance or participation. Bipartite graphs find applications in various domains, including recommendation systems, where they help identify connections between users and items for personalized recommendations, as well as in matching problems, where they can be used to optimize pairings or assignments while ensuring compatibility. The simplicity and versatility of bipartite graphs make them a valuable tool for understanding and analyzing diverse real-world relationships.

Need of the Study

The study on exploring complex networks through multidimensional analysis addresses a pressing need in various fields, including social sciences, biology, information technology, and transportation systems. Complex networks are ubiquitous in our interconnected world, representing intricate relationships and interactions among entities, such as individuals in social networks, proteins in biological systems, or nodes in the internet. Traditional network analysis methods often fall short in capturing the full complexity of these systems. Multidimensional analysis offers a powerful solution by considering multiple attributes, such as node attributes, temporal dynamics, and community structures simultaneously. This approach enables a more comprehensive understanding of network behaviors and functionalities. The need for this study arises from the increasing volume and diversity of data available in today's digital age. Multidimensional analysis can help reveal hidden patterns, identify critical nodes, detect anomalies, and optimize network performance. In social sciences, it can aid in understanding the dynamics of online communities or the spread of information and misinformation. In biology, it can assist in uncovering intricate regulatory networks within cells. Exploring complex networks through multidimensional analysis is crucial for advancing our understanding of complex systems and making informed decisions in various domains. This study seeks to provide valuable insights and tools to tackle the challenges posed by modern interconnected networks.

Critical analysis of complex networks

Critical analysis of complex networks is crucial for advancing our understanding of various systems, from social networks to biological networks and infrastructure systems. One key aspect of such analysis is the recognition that not all networks exhibit the same properties, and applying one-size-fits-all approaches may lead to oversimplification. The limitations of available data can hinder analysis and result in biased or incomplete insights. Furthermore, the interpretation of network metrics and structures must be context-dependent, as the same pattern in different networks may carry different meanings. Additionally, complex networks often involve dynamic processes, necessitating the incorporation of temporal and multidimensional perspectives. Critical analysis should also address ethical concerns, such as privacy and security, particularly in social and technological networks. Critical analysis of complex networks requires a thoughtful and context-aware approach, acknowledging the diversity and dynamic nature of these systems, while considering the ethical implications of network research.

Static robustness

Static robustness is a critical consideration in various domains, encompassing networks, cybersecurity, infrastructure, and supply chain management. It reflects a system's ability to withstand disruptions, failures, or attacks without considering dynamic adaptations over time. For example, in network science, a statically robust network can maintain its functionality even when certain nodes or connections are compromised. Similarly, software systems and critical infrastructure must exhibit static robustness to resist vulnerabilities and threats. However, it's essential to remember that static robustness is just one aspect of overall system resilience. Real-world scenarios often demand dynamic responses and adaptive strategies, emphasizing the need to complement static assessments with dynamic analysis. While static robustness assessment identifies weaknesses, a comprehensive approach also considers how a system can recover, adapt, or reconfigure itself when confronted with adverse conditions, ensuring its ability to thrive in a constantly changing environment.

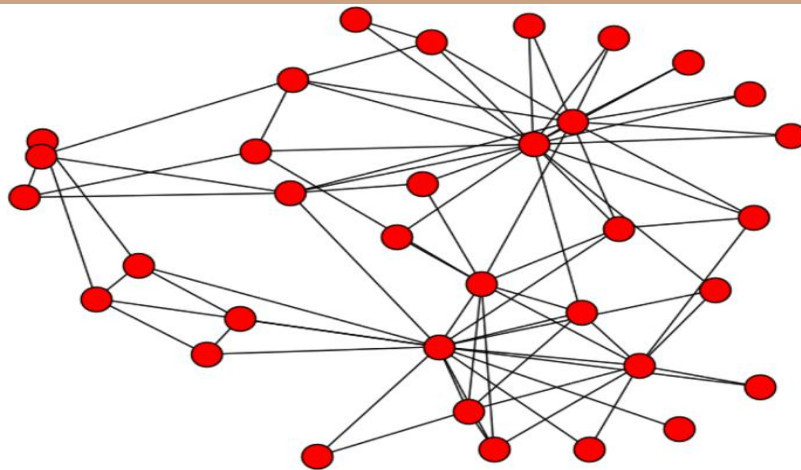
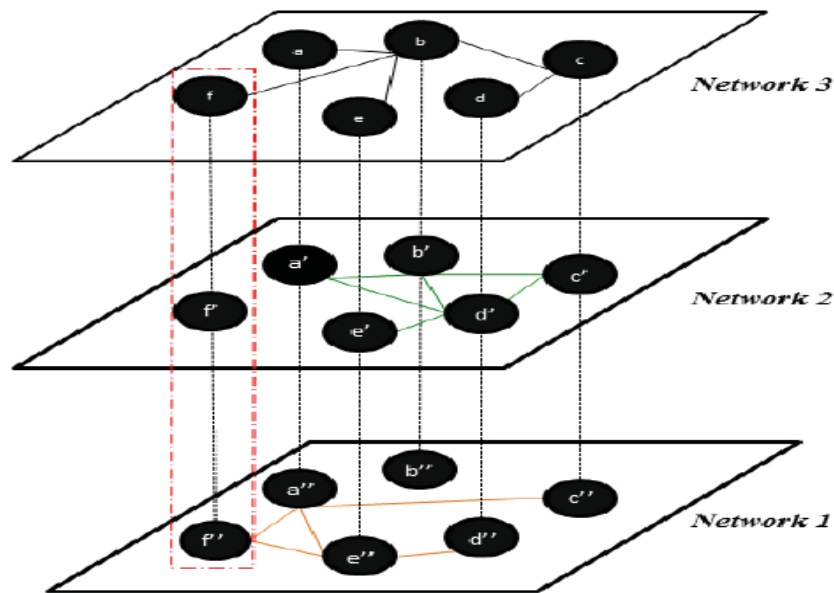


Fig 4 Complex Networks

Complex networks are intricate and often non-trivial structures that play a fundamental role in understanding the interconnected nature of various systems. These networks are prevalent in our daily lives, spanning social interactions, the World Wide Web, biological systems, transportation networks, and more. What makes them intriguing are their unique properties, including scale-free degree distributions, small-world phenomena, and community structures, which deviate from traditional regular networks. Researchers in diverse fields leverage complex network analysis to uncover hidden patterns, predict behaviors, and optimize systems. By studying the topological and dynamic properties of these networks, we gain insights into how information, disease, or influence spreads, identify critical nodes or hubs, and enhance network resilience. Complex networks research continues to thrive as it bridges disciplines and offers valuable tools to address complex problems in a wide range of domains.

Multidimensional networks

Multidimensional networks provide a versatile and insightful approach to modeling the intricate relationships and interactions within complex systems. By accommodating multiple layers and interlayer connections, they offer a more comprehensive representation of real-world phenomena across various contexts. These networks find applications in a wide range of fields, from transportation and urban planning to neuroscience and biology, allowing researchers to explore and analyze complex systems with greater fidelity. The integration of temporal dynamics and the study of resilience in multidimensional networks contribute to a deeper understanding of how disruptions and changes impact the overall system.



However, the complexity of multidimensional network visualization and community detection poses unique challenges that researchers are actively addressing. Overall, multidimensional networks are a valuable tool for unraveling the complexities of interconnected systems and gaining insights that can inform decision-making and optimization in diverse domains.

Degree Related Measures

Definition 1 (Degree):

In the context of a network represented by the set V of nodes and the set E of edges, where D is a set of possible attributes or dimensions, the degree of a node $v \in V$ with respect to a subset of dimensions D_0 (where $D_0 \subseteq D$) is defined as follows:

$\text{Degree}(v, D_0) =$ The number of edges $(u, v, d) \in E$ such that u is a node in V and d is an attribute or dimension in D_0 .

In simpler terms, the degree of node v with respect to dimensions D_0 counts the number of edges that connect v to other nodes in the network, considering only those edges that exist within the specified subset of dimensions D_0 . This concept is fundamental in network analysis and helps quantify the connectivity or importance of a node within specific dimensions or attributes of the network.

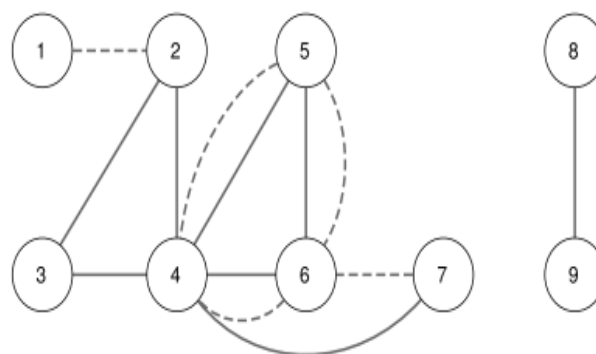


Fig 5 The solid line is dimension 1, the dashed line is dimension 2.

Definition 2 (Average of the Degrees over dimensions):

In the context of a network represented by the set V of nodes and the set D of dimensions (attributes), the average degree of a node $v \in V$ with respect to a subset of dimensions D_0 (where $D_0 \subseteq D$) is defined as follows:

$$\text{AvgDegree}(v, D_0) = \text{Degree}(v, D_0) / |D_0|$$

Here, $\text{Degree}(v, D_0)$ represents the degree of node v with respect to the dimensions in D_0 , as defined in Definition 1. The average degree is computed by dividing this degree by the number of dimensions in the subset D_0 .

In essence, $\text{AvgDegree}(v, D_0)$ quantifies the average connectivity of node v within the specified set of dimensions D_0 in the network G . This metric provides insights into how connected or influential a node is across a specific set of attributes.

Example

In Example, we consider a multigraph that represents a multidimensional network with two dimensions: dimension d_1 , represented by a solid line, and dimension d_2 , represented by a dashed line. Let's calculate the degree and average degree of node 3 with respect to these dimensions:

- $\text{Degree}(3, \{d_1\}) = 2$: Node 3 has 2 edges in dimension d_1 .
- $\text{Degree}(3, \{d_2\}) = 0$: Node 3 does not have any edges in dimension d_2 .
- $\text{AvgDegree}(3, \{d_1, d_2\}) = (2 + 0) / 2 = 1$: The average degree of node 3 considering both dimensions d_1 and d_2 is 1, as there are 2 edges in total, and we divide by the number of dimensions (2) to get the average.

- $\text{AvgDegree}(3, \{d1\}) = 2 / 1 = 2$: The average degree of node 3 considering only dimension $d1$ is 2, as there are 2 edges in dimension $d1$, and we divide by the number of dimensions (1) to get the average.

These calculations illustrate how the degree and average degree metrics are applied to a multidimensional network, considering different dimensions, to quantify the connectivity and average connectivity of a node within specific attribute sets

Applications

The application of complex networks through multidimensional analysis spans various domains, as this approach provides a powerful framework for understanding and addressing complex problems.

Here are some key applications:

1. **Social Networks Analysis:** Multidimensional analysis of social networks considers various types of interactions (e.g., friendships, collaborations, communication) among individuals. It helps in understanding the structure of social communities, information diffusion, and influence dynamics, aiding in marketing, recommendation systems, and social sciences research.
2. **Transportation Systems:** In transportation, multidimensional analysis can model interactions among commuters, modes of transport (e.g., road, rail, air), and infrastructure layers. It improves traffic management, optimizes route planning, and enhances urban mobility.
3. **Biology and Neuroscience:** Multidimensional networks are employed to study complex biological systems where different layers represent interactions among genes, proteins, metabolic pathways, and more. In neuroscience, they help uncover brain connectivity patterns across various cognitive functions and brain regions.
4. **Epidemiology and Healthcare:** Understanding disease transmission requires considering various contact types and transmission routes. Multidimensional networks are crucial in modeling infectious disease spread and evaluating public health interventions.
5. **Recommendation Systems:** In e-commerce and content recommendation, multidimensional analysis enables personalized recommendations by considering user-product interactions, user preferences, and contextual factors to enhance user satisfaction and engagement.

6. **Smart Cities and Urban Planning:** Urban planners use multidimensional networks to integrate data from various sources, such as transportation, energy consumption, and social interactions, to optimize city operations, improve infrastructure, and enhance urban sustainability.
7. **Supply Chain Management:** Multidimensional networks help model complex supply chain networks with multiple tiers, suppliers, and products. This analysis improves supply chain resilience, efficiency, and risk management.
8. **Online Social Platforms:** Social media platforms employ multidimensional analysis to understand user engagement, content sharing, and user behavior across multiple dimensions. This informs content recommendation and platform design.
9. **Protein-Protein Interaction Networks:** In bioinformatics, multidimensional networks represent interactions among proteins, genes, and biological pathways. This aids in drug discovery, understanding disease mechanisms, and designing targeted therapies.
10. **Financial Systems:** In finance, multidimensional networks can represent relationships among financial institutions, assets, and risk factors, enhancing risk assessment, portfolio optimization, and systemic risk analysis.
11. **Energy Grids:** In the energy sector, multidimensional analysis helps model interactions among power generation sources, consumers, and the grid infrastructure to optimize energy distribution, manage demand, and enhance grid reliability.

In these applications and more, multidimensional analysis of complex networks provides valuable insights, enabling informed decision-making, optimization, and improved understanding of the intricate relationships within complex systems.

Real World Multidimensional Networks

Facebook

This network represents a small ego-centered subset extracted from a well-known social media platform. It was meticulously constructed by considering the immediate connections of the author of this thesis. Specifically, we included the direct friends of the author and then established connections among these friends while excluding the author, whose connections naturally span all

individuals, thus causing interference in the analysis. This network comprises a total of 228 nodes and over 3,000 edges, as indicated in Table, which provides additional topological statistics.

Our approach to creating this network involves ten different dimensions. These dimensions encompass various aspects, including direct friendships, shared affiliations with common groups or events, instances of co-appearance in photos, mutual engagement through comments or "likes" (utilizing the social media platform's functions) related to a specific object, or the joint tagging of individuals in the same message or video. Notably, all dimensions, except for direct friendships, employ a "tf-idf" (term frequency-inverse document frequency) approach. This means that groups or events that are excessively popular within this network are downweighted, discouraging the formation of connections in those particular dimensions.

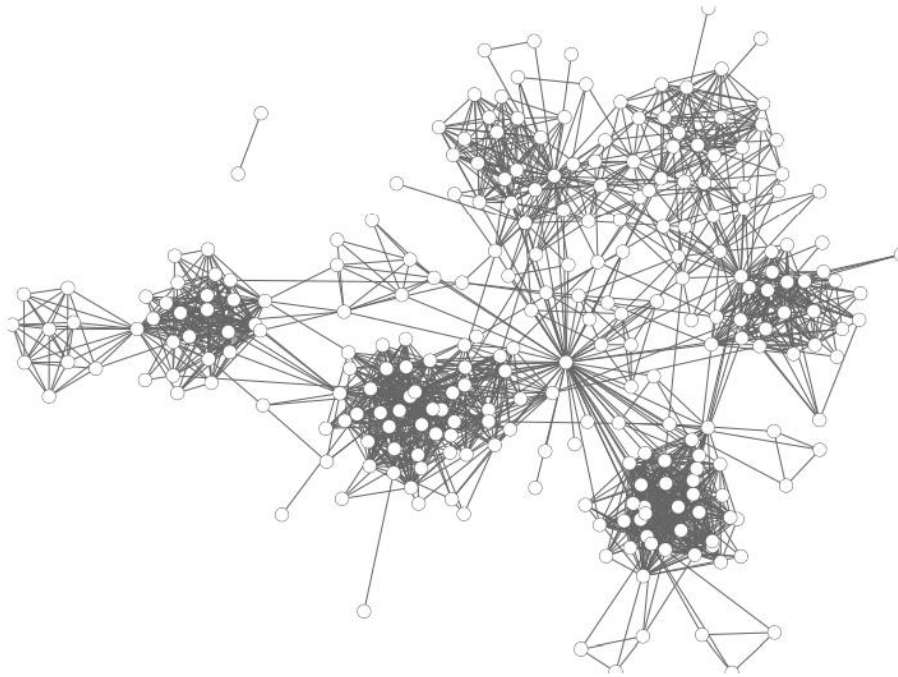


Figure 6: The friendship dimension in Facebook network.

Supermarket

This network originates from sales data collected from a chain of Italian supermarkets, with its nodes representing individual supermarket customers. Nodes in this network are connected to each other based on their shared buying behavior. Initially, the dataset consisted of a substantial 90 million original transactions over the course of a year, involving 838,000 unique customers and 318,000 distinct products. To structure the data, the supermarket's 732 marketing categories for products were utilized for clustering purposes.

Table 1 Main statistics about Facebook and Supermarket networks, and their dimensions.

Dataset	Dimension	<i>n</i>	<i>m</i>	<i>k</i>
Facebook	Friendship	225	1,371	12.186
	Group	118	494	8.372
	Comment	64	92	2.875
	Likes	83	337	8.120
	Photo Tag	154	439	5.701
	Status Comment	133	236	3.548
	Status Tag	14	15	2.142
	Video Tag	17	18	2.117
	Event	88	259	5.886
	Note Comment	48	50	2.083
	Global	228	3,311	29.043
Supermarket	Mozzarella Cheese	1,578	8,162	10.344
	Bread	1,749	6,912	7.903
	Clementine	1,101	4,061	7.376
	Bananas	1,291	5,282	8.182
	Short Pasta	1,420	6,741	9.494
	Red Meat	1,329	5,081	7.646
	Canned Vegetables	1,320	4,808	7.284
	Long Pasta (Spaghetti)	1,312	5,187	7.907
	Milk UHT	1,665	7,202	8.651
	Mineral Water	1,998	12,141	12.153
	Global	4,463	65,577	29.386

For the creation of this specific network, a subset was selected, encompassing a two-week period, 4,000 randomly chosen customers, and 10 marketing categories, which serve as our dimensions of analysis. The resulting network offers insights into customer behavior patterns and purchasing preferences within the specified timeframe and product categories.

when we mention that two customers share the same buying behavior, we are not referring to the quantity or volume of a specific product they purchase. Instead, we are assessing the similarity in terms of the relative importance of a particular product within the context of each customer's overall shopping preferences. To quantify this similarity, we employ a specific measure known as the Revealed Comparative Advantage (RCA).

$$RCA(c, p) = \frac{\left(\frac{x(c, p)}{\sum_p x(c, p)} \right)}{\left(\frac{\sum_c x(c, p)}{\sum_{c, p} x(c, p)} \right)},$$

The Revealed Comparative Advantage (RCA) is calculated using the formula:

$$RCA(c, p) = (x(c, p) / X(c)) / (X(p) / N)$$

Here, $x(c, p)$ represents the total amount purchased by customer c of product p . $RCA(c, p)$ is greater than one when the proportion of purchases made by customer c for a particular product p is higher than the proportion of that product's sales in the entire supermarket's purchases. In essence, it signifies that the customer has a higher preference or inclination towards that specific product compared to the average shopper in the supermarket. This measure is conceptually equivalent to the "lift," a well-known concept in association rules mining, which quantifies the strength of association between items in a dataset.

$$\phi(i, j, p) = \min \left\{ \frac{RCA(i, p)}{RCA(j, p)}, \frac{RCA(j, p)}{RCA(i, p)} \right\}.$$

The function $\phi(i, j, p)$ serves as a similarity measure between customers i and j with respect to product p . It takes on a value of 1 when the Revealed Comparative Advantages (RCAs) of customers i and j for product p are identical, indicating a strong similarity in their preferences for that specific product. As the RCAs of i and j for product p diverge, $\phi(i, j, p)$ gradually decreases towards zero, signifying diminishing similarity.

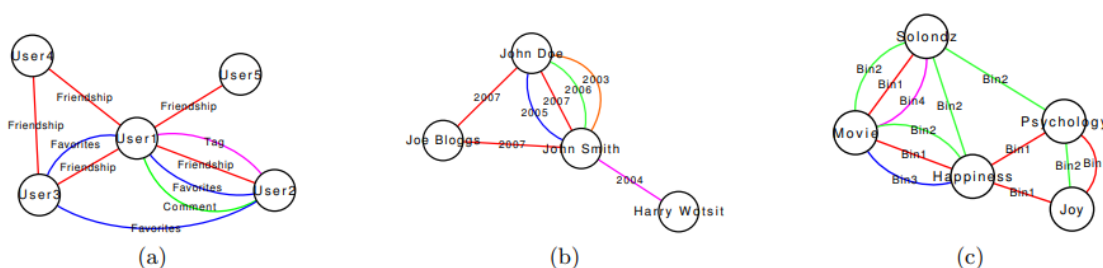


Figure 7: Small extracts of the three real multidimensional networks.

Customers at a supermarket might have distinctive preferences, choosing to buy specific products exclusively from one store while opting for different items at other shops. For instance, someone may have a preference for purchasing meat products solely from a nearby store rather than the supermarket's meat department. In such cases, we can apply a strategy of grouping closely connected nodes across multiple redundant dimensions. By doing so, we can identify specific dimensions where these groups of customers unexpectedly lack connections.

Research Problem

Exploring complex networks through multidimensional analysis offers a multitude of exciting research problems and avenues. Researchers in this field grapple with the challenge of developing

scalable algorithms capable of handling massive multidimensional networks. Community detection in such networks, accounting for interlayer connections and temporal dynamics, remains a complex problem. Additionally, understanding the impact of disruptions and failures in one layer on the overall system's resilience is critical for optimizing system robustness. Effective visualization techniques for multidimensional networks are in demand to convey insights clearly. Integrating data from various sources while ensuring data quality, addressing privacy and security concerns, and tailoring analysis techniques to domain-specific applications all present intriguing challenges. Furthermore, interdisciplinary collaboration and ethical considerations play vital roles in advancing our understanding of complex systems through multidimensional network analysis. Tackling these research problems promises to deepen our insights into the complex fabric of interconnected systems and provide solutions to real-world challenges across diverse domains.

Conclusion

Exploring complex networks through multidimensional analysis is a dynamic and multifaceted field that continues to evolve and expand our understanding of complex systems across various domains. This approach allows us to delve deeper into the intricate relationships, interactions, and structures that define these systems. Through this journey of exploration, we encounter a range of research problems and challenges that span scalability, community detection, temporal dynamics, resilience, visualization, data integration, privacy, machine learning integration, domain-specific applications, interdisciplinary collaboration, and ethical considerations. Addressing these challenges is crucial for unlocking the full potential of multidimensional network analysis. As researchers and practitioners, we are poised to make significant strides in optimizing complex systems, from urban planning and transportation networks to healthcare, finance, and beyond. By developing innovative algorithms, visualization techniques, and ethical frameworks, we can harness the power of multidimensional analysis to drive meaningful insights, make informed decisions, and navigate the complex web of interactions that shape our world.

References

1. Alain Barrat, Marc Barthlemy, and Alessandro Vespignani. *Dynamical Processes on Complex Networks*. Cambridge University Press, New York, NY, USA, 2008
2. Possamai, L. (2013). *Multidimensional analysis of complex networks*.
3. Costa, L. D. F., Rodrigues, F. A., Travieso, G., & Villas Boas, P. R. (2007). *Characterization of complex networks: A survey of measurements*. *Advances in physics*, 56(1), 167-242.

4. Gao, Z. K., Small, M., & Kurths, J. (2017). Complex network analysis of time series. *Europhysics Letters*, 116(5), 50001.
5. Haraguchi, Y., Shimada, Y., Ikeguchi, T., & Aihara, K. (2009). Transformation from complex networks to time series using classical multidimensional scaling. In *Artificial Neural Networks–ICANN 2009: 19th International Conference, Limassol, Cyprus, September 14-17, 2009, Proceedings, Part II 19* (pp. 325-334). Springer Berlin Heidelberg.
6. Araújo, T., & Banisch, S. (2016). Multidimensional analysis of linguistic networks. In *Towards a Theoretical Framework for Analyzing Complex Linguistic Networks* (pp. 107-131). Berlin, Heidelberg: Springer Berlin Heidelberg.
7. Kumar, S., Panda, B. S., & Aggarwal, D. (2021). Community detection in complex networks using network embedding and gravitational search algorithm. *Journal of Intelligent Information Systems*, 57, 51-72.
8. Liu, S., Gao, X., Fang, W., Sun, Q., Feng, S., Liu, X., & Guo, S. (2018). Modeling the complex network of multidimensional information time series to characterize the volatility pattern evolution. *IEEE Access*, 6, 29088-29097.
9. Gao, Z. K., Yang, Y. X., Fang, P. C., Zou, Y., Xia, C. Y., & Du, M. (2015). Multiscale complex network for analyzing experimental multivariate time series. *Europhysics Letters*, 109(3), 30005.
10. Arunkumar, A. T., & Ambrose, M. Multidimensional analysis of complex networks.
11. Cencetti, G., Bagnoli, F., Battistelli, G., Chisci, L., & Fanelli, D. (2017). Control of multidimensional systems on complex network. *Plos one*, 12(9), e0184431.
12. Gao, Z., Dang, W., Wang, X., Hong, X., Hou, L., Ma, K., & Perc, M. (2021). Complex networks and deep learning for EEG signal analysis. *Cognitive Neurodynamics*, 15, 369-388.
